Vision Algorithms for Mobile Robotics

Lecture 05
Point Feature Detection and Matching – Part 1

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Lab Exercise 3 - Today

Implement the Harris corner detection and matching

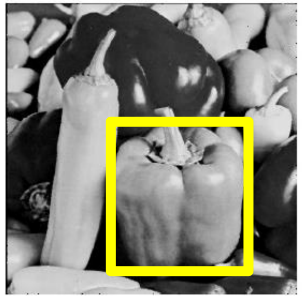
Outline

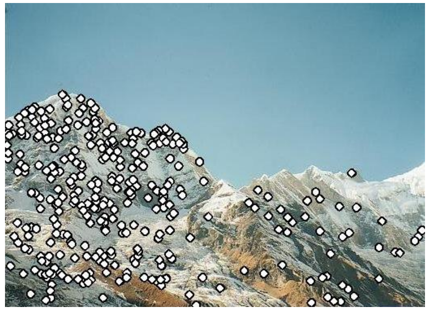
- Filters for Feature detection
- Point-feature extraction: today and next lecture

Filters for Feature Detection

- In the last lecture, we used filters to reduce **noise** or enhance **contours** (i.e., edge detection)

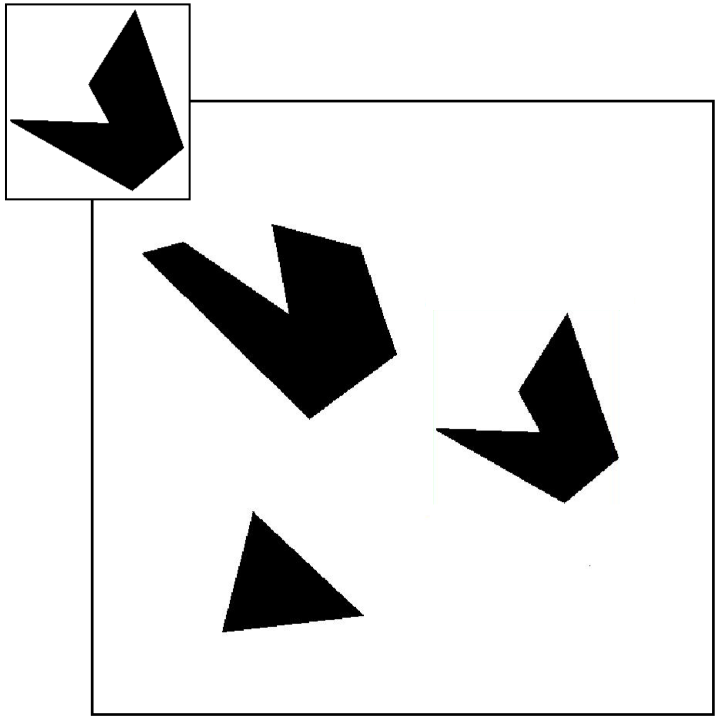
- However, filters can also be used to detect "**features**"
  Goal: reduce amount of data to process in later stages, discard redundancy to preserve only what is useful (leads to **lower bandwidth and memory storage**)
  - **Edge detection** (we have seen this already; edges can enable line or shape detection)
  - **Template matching**
  - **Keypoint detection**



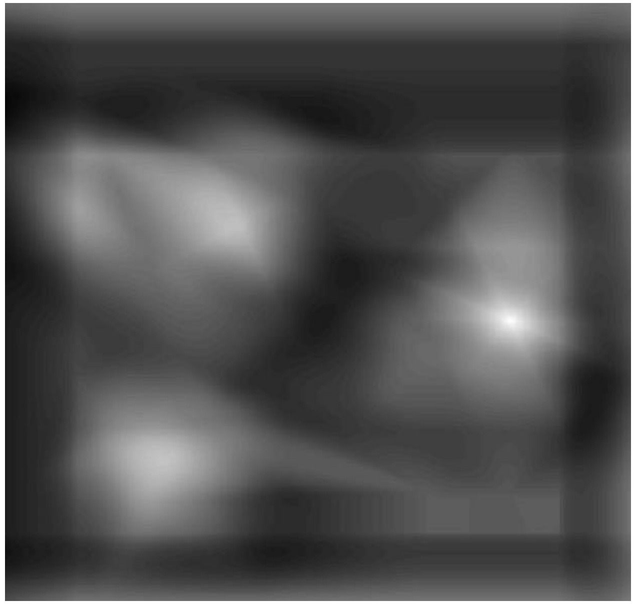


Filters for Template Matching

- Find locations in an image I that are similar to a ***template*** H
- If we look at filters as **templates**, we can use **cross-correlation** (see lecture 4, like convolution but without rotating the filter) to detect these locations



H



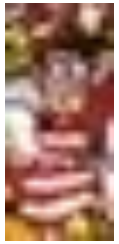
Correlation map I':

$$I'[x,y] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} I[x+u, y+v] H[u,v]$$

I

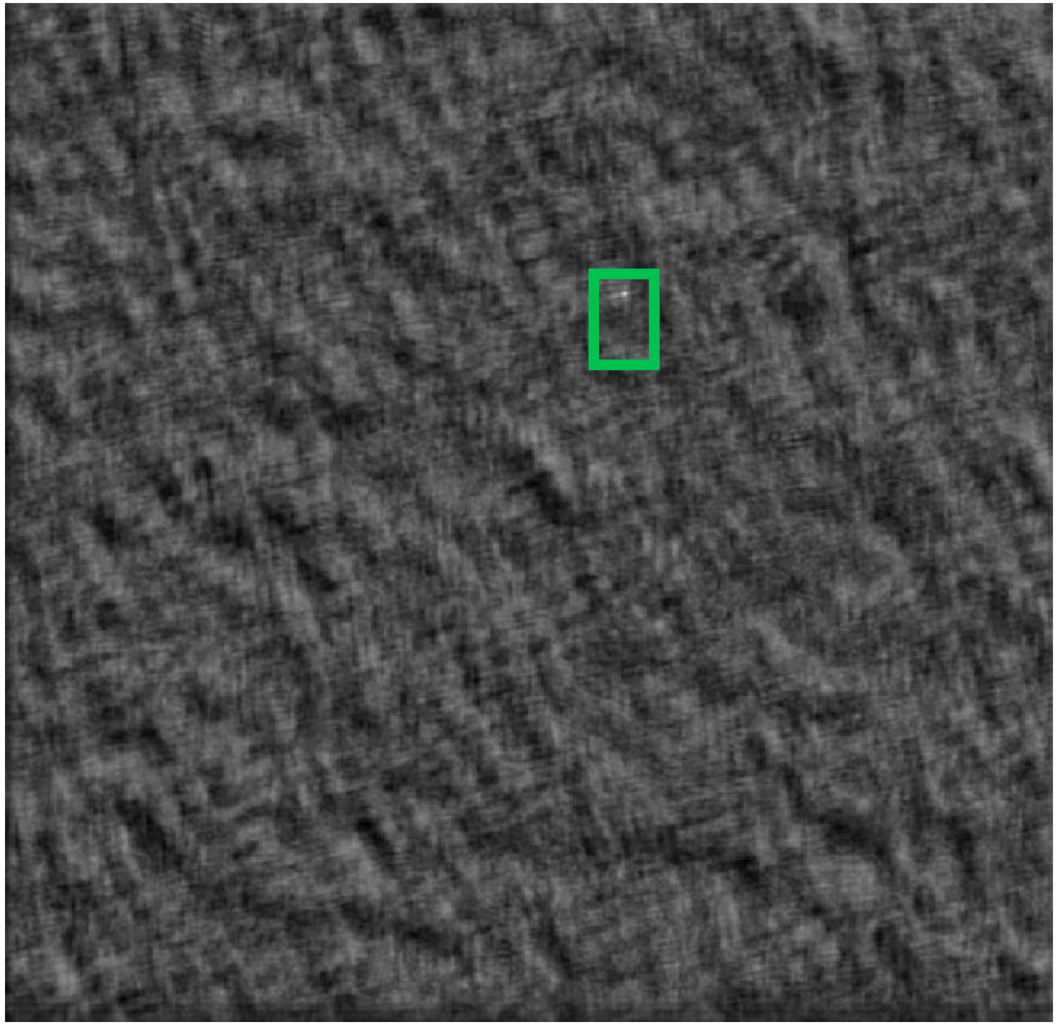
Where’s Waldo?

Scene

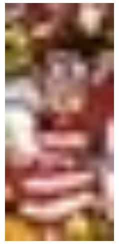


Template

Where's Waldo?



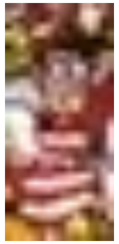
Scene



Template

Where's Waldo?

Scene



Template

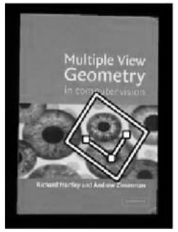
Template Matching

- What if the template is not identical to the object we want to detect?
- What about the pixels in the template's background (object-background problem)?
- Template Matching will only work if **scale**, **orientation**, **illumination**, and, in general, the **appearance of the template** (including anything in background) and the object to detect are **very similar**.

Scene

Template

Scene

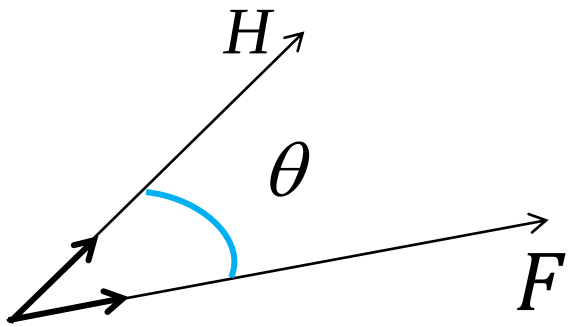


Template

Correlation as Scalar Product

- Consider two image patches H and F of same size as 1-dimensional vectors with n entries (where n is the number of pixels), their cross-correlation can be written as an inner product:

$$\langle H, F \rangle = \|H\| \|F\| \cos \theta$$



- In **Normalized Cross Correlation (NCC)**, we consider the unit vectors of H and F, hence we measure their similarity based on the angle θ.

$$\cos \theta = \frac{\langle H, F \rangle}{\|H\| \|F\|}$$

$$NCC = \frac{\sum_{u=-k}^{k} \sum_{v=-k}^{k} H(u,v) F(u,v)}{\sqrt{\sum_{u=-k}^{k} \sum_{v=-k}^{k} H(u,v)^2} \sqrt{\sum_{u=-k}^{k} \sum_{v=-k}^{k} F(u,v)^2}}$$

Similarity Measures

- **Normalized Cross Correlation** (**NCC**): ranges between -1 and +1 and is exactly 1 if H and F are identical

$$NCC = \frac{\sum_{u=-k}^{k}\sum_{v=-k}^{k} H(u,v)F(u,v)}{\sqrt{\sum_{u=-k}^{k}\sum_{v=-k}^{k} H(u,v)^2}\sqrt{\sum_{u=-k}^{k}\sum_{v=-k}^{k} F(u,v)^2}}$$

- **Sum of Squared Differences** (**SSD**): awalys ≥ 0. It's exactly 0 only if H and F are identical

$$SSD = \sum_{u=-k}^{k}\sum_{v=-k}^{k}\left(H(u,v) - F(u,v)\right)^2$$

- **Sum of Absolute Differences** (**SAD**) (used in optical mice): awalys ≥ 0. It's 0 only if H and F are identical

$$SAD = \sum_{u=-k}^{k}\sum_{v=-k}^{k}\left|H(u,v) - F(u,v)\right|$$

Zero-mean SAD, SSD, NCC

To account for the difference in the average intensity of two images (typically caused by additive illumination changes), we subtract the mean value of each image:

$$\mu_H = \frac{1}{n}\sum_{u=-k}^{k}\sum_{v=-k}^{k} H(u,v) \qquad \mu_F = \frac{1}{n}\sum_{u=-k}^{k}\sum_{v=-k}^{k} F(u,v)$$

n is the number of pixels of H or F

- **Zero-mean Normalized Cross Correlation (ZNCC)**

$$ZNCC = \frac{\sum_{u=-k}^{k}\sum_{v=-k}^{k}(H(u,v)-\mu_H)(F(u,v)-\mu_F)}{\sqrt{\sum_{u=-k}^{k}\sum_{v=-k}^{k}(H(u,v)-\mu_H)^2}\sqrt{\sum_{u=-k}^{k}\sum_{v=-k}^{k}(F(u,v)-\mu_F)^2}}$$

ZNCC is invariant to affine intensity changes: $I'(x,y) = \alpha I(x,y) + \beta$

- **Zero-mean Sum of Squared Differences (ZSSD)**

$$ZSSD = \sum_{u=-k}^{k}\sum_{v=-k}^{k}\left((H(u,v)-\mu_H)-(F(u,v)-\mu_F)\right)^2$$

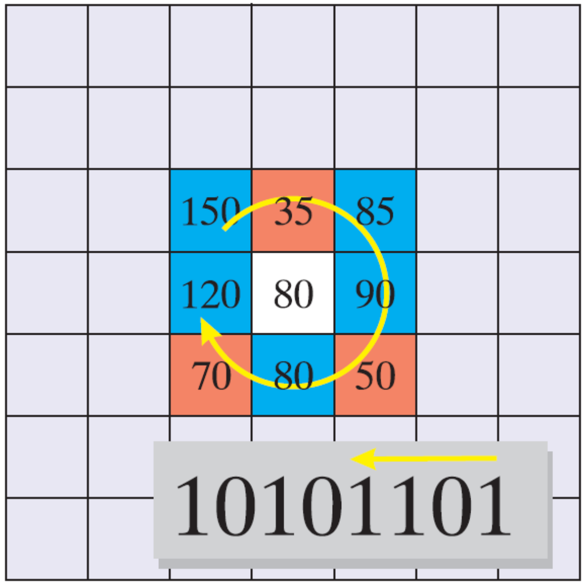
- **Zero-mean Sum of Absolute Differences (ZSAD)** (used in optical mice)

$$ZSAD = \sum_{u=-k}^{k}\sum_{v=-k}^{k}\left|(H(u,v)-\mu_H)-(F(u,v)-\mu_F)\right|$$

Are these invariant to affine illumination changes?

Census Transform & Hamming Distance

- Maps an image patch to a bit string:
  - **if a pixel intensity is greater than or equal to the center pixel intensity**, its corresponding bit is set to **1**, else to 0
  - For a $w \times w$ patch, the string will be $\boldsymbol{w^2 - 1}$ **bits long**
- The two bit strings are **compared using the Hamming distance**, which is the number of bits that are different. This can be computed by counting the number of 1s in the Exclusive-OR (XOR) of the two bit strings



Advantages

- **No square roots or divisions** are required, thus very efficient to implement, especially on FPGA
- Intensities are considered relative to the center pixel of the patch making it **invariant to monotonic nonlinear intensity** changes

Outline

- Filters for Feature detection
- Point-feature extraction: today and next lecture

Keypoint extraction and matching - Example

Video from "Forster, Pizzoli, Scaramuzza, *SVO: Semi-Direct Visual Odometry*". IEEE Transactions on Robotics, 2017. PDF. Video

Why do we need keypoints?

- Recall the Visual-Odometry flow chart:

Image sequence
↓
Feature detection
↓
Feature matching (or tracking)
↓
Motion estimation
↓
Local optimization

Features tracked over multiple recent frames overlaid on the last frame

Why do we need keypoints?

- Keypoint extraction is the key ingredient of motion estimation

Image sequence

↓

Feature detection

↓

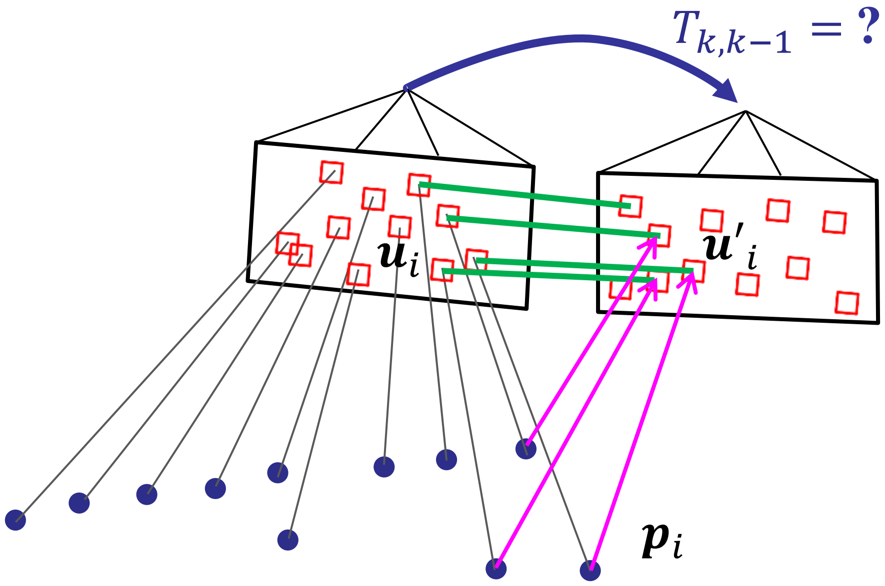
Feature matching (or tracking)

↓

Motion estimation

↓

Local optimization

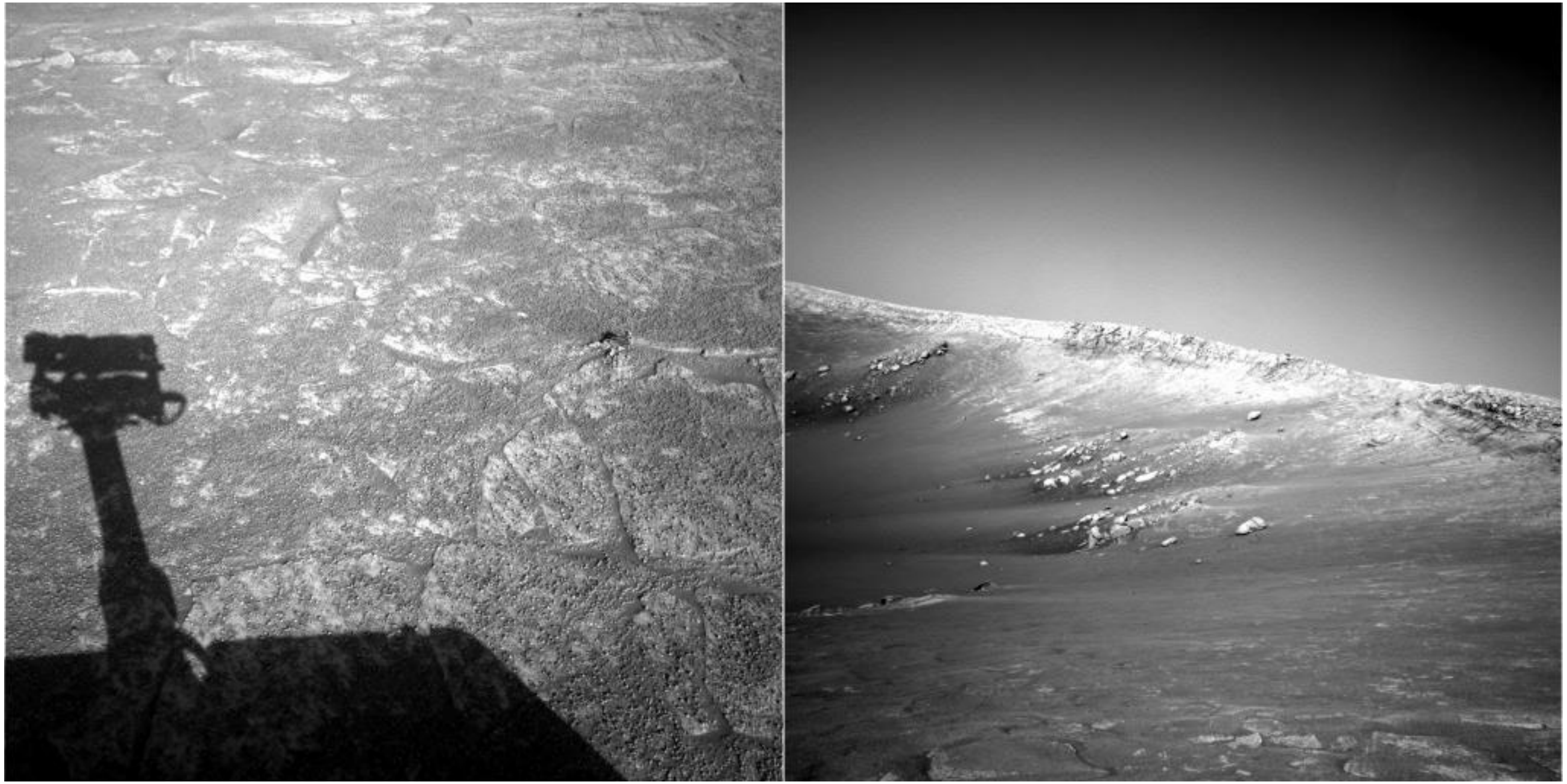


Keypoints are also used for:

- Panorama stitching
- Object recognition
- 3D reconstruction
- Place recognition
- Indexing and database retrieval (e.g., Google Images or http://tineye.com)

- These problems go under the name of **Feature Matching problem:** finding similar keypoints between two images of the same scene taken under different conditions

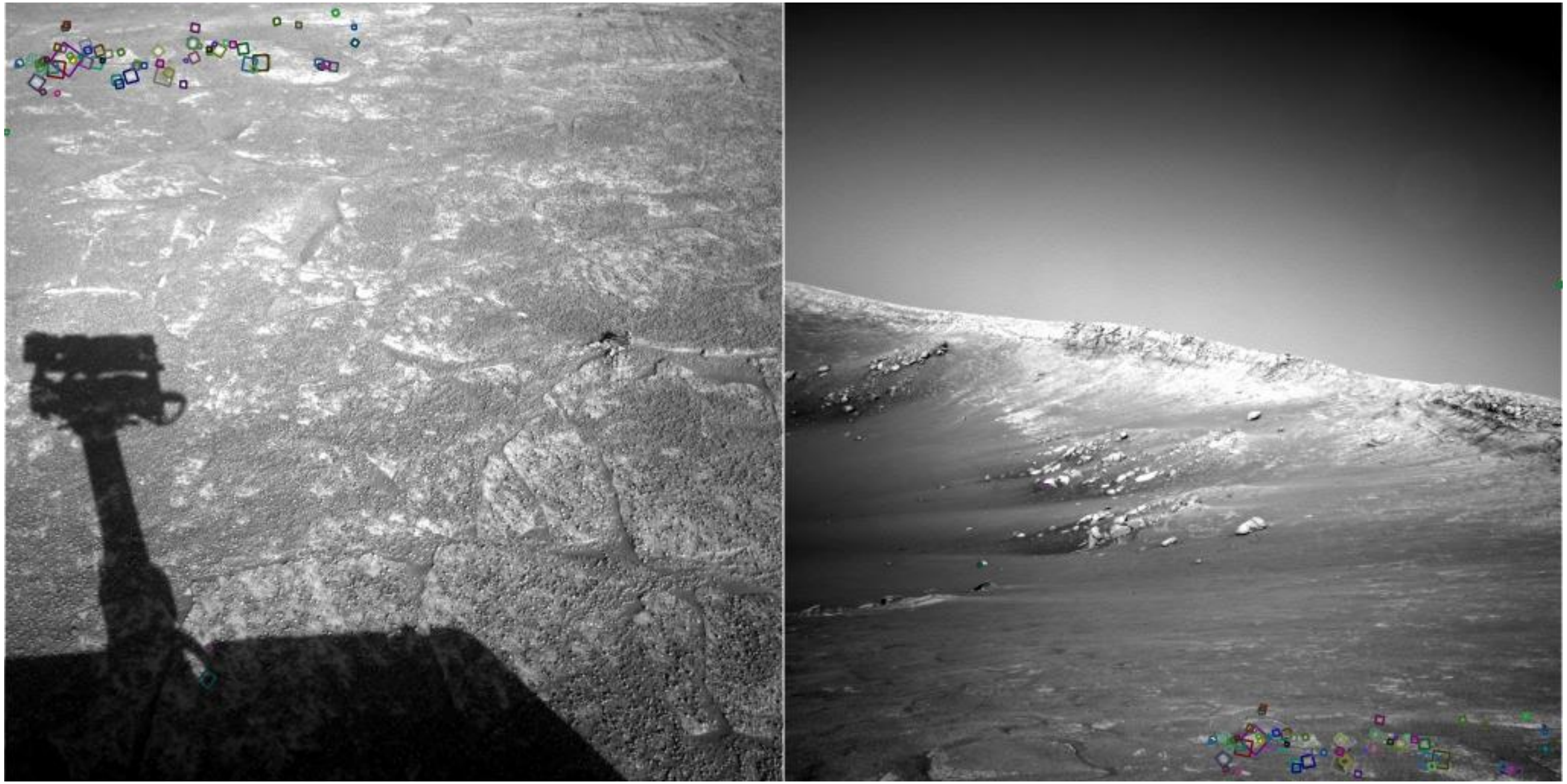
Image matching: why is it challenging?



NASA Mars Rover images

Image matching: why is it challenging?

- Answer below



NASA Mars Rover images with SIFT feature matches

Example: panorama stitching

How does it work?

AutoStitch: http://matthewalunbrown.com/autostitch/autostitch.html
M. Brown and D. G. Lowe. Recognising Panoramas, International Conference on Computer Vision (ICCV), 2003. PDF.

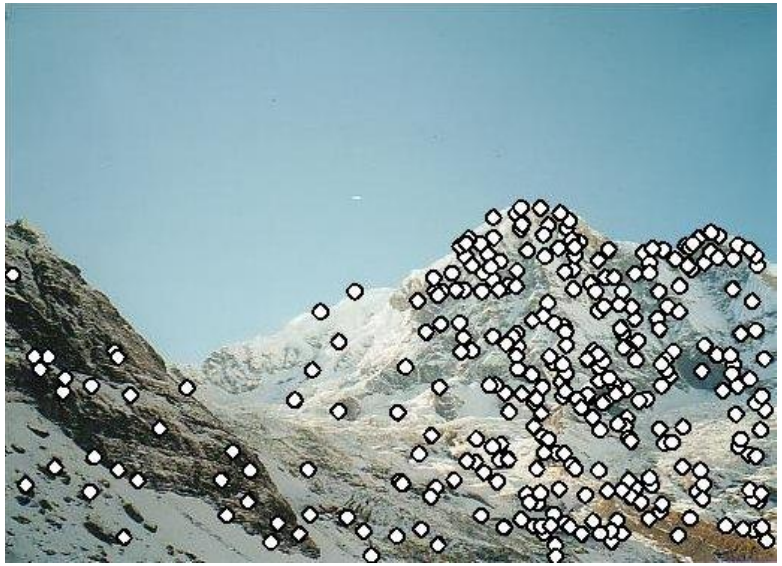
Local features and alignment

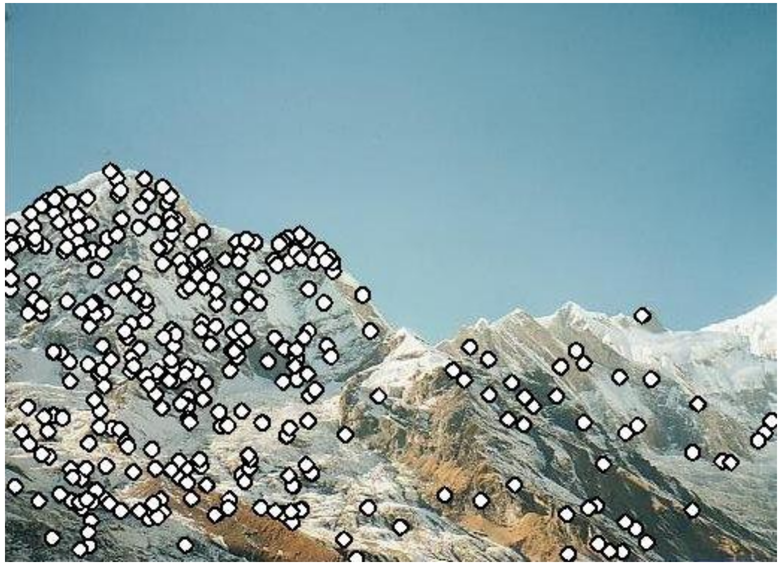
- We need to align two images
- How would you do it?

Local features and alignment

Idea:

- Detect point features in both images

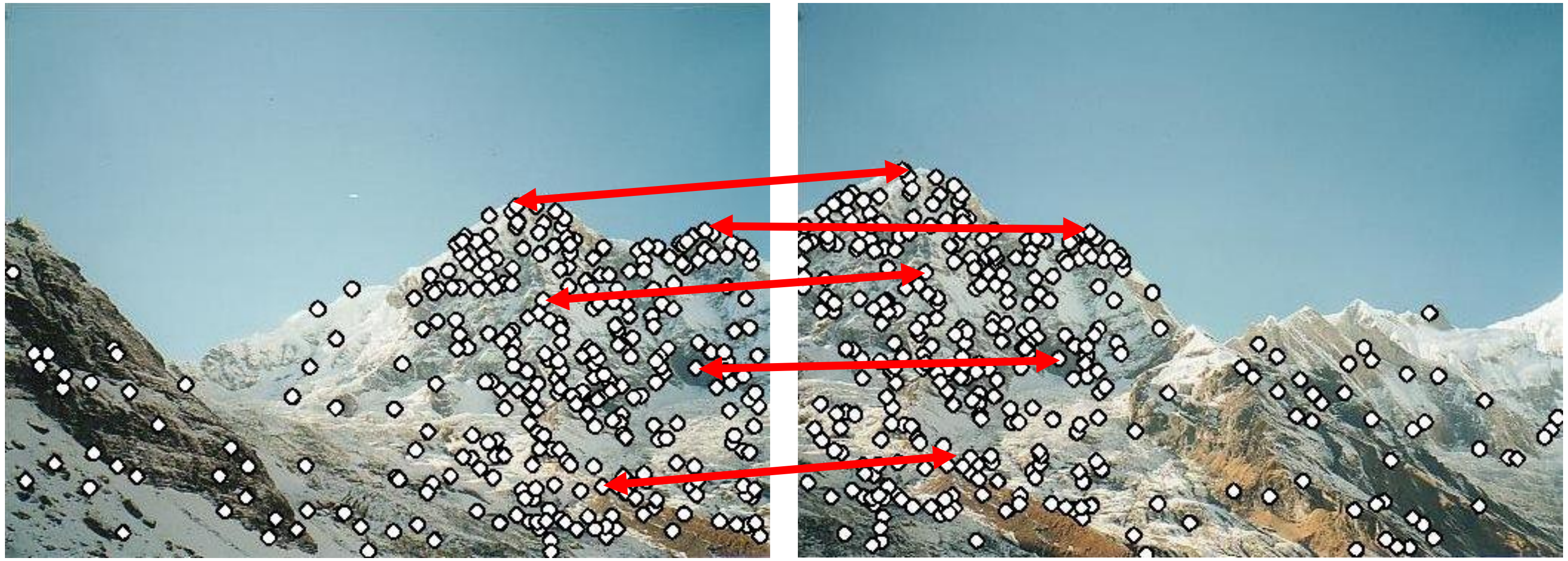




Local features and alignment

Idea:

- Detect point features in both images
- Find corresponding pairs



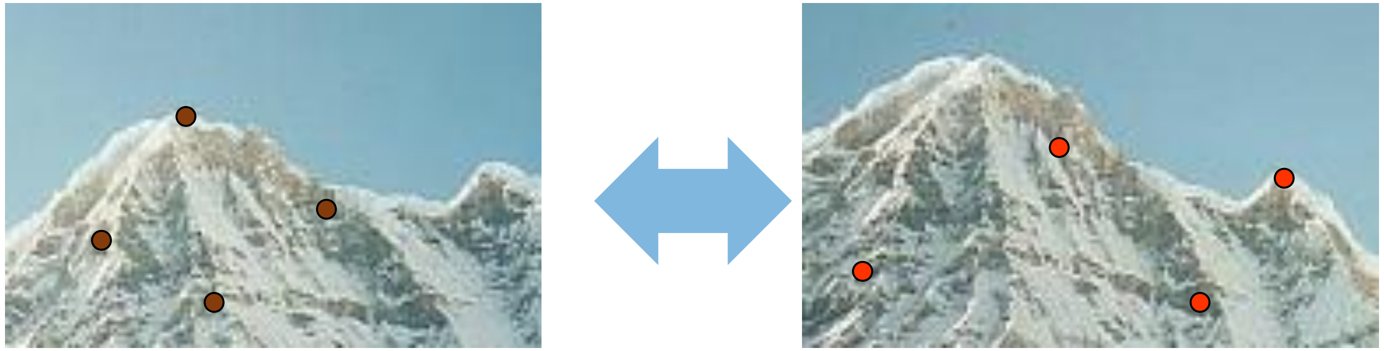
Local features and alignment

Idea:

- Detect point features in both images
- Find corresponding pairs
- Use these pairs to align the images: what image transformation would you use?

Matching with Features

Problem 1: How to **detect** the **same** points **independently** in both images?



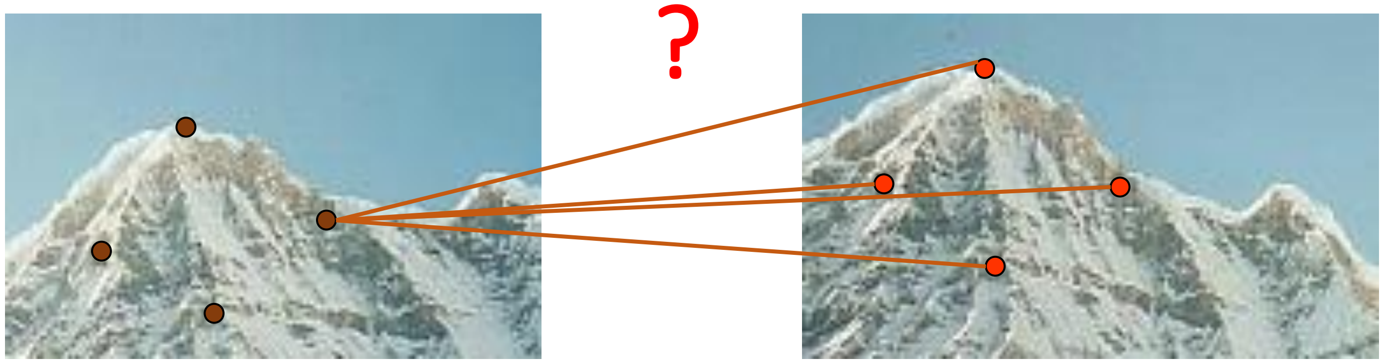
no chance to match!

We need a **repeatable** feature **detector.** Repeatable means that the detector should be able to re-detect the same feature in different images of the same scene, so it should be **robust to geometric and photometric** changes.

This property is called **Repeatability** of a feature **detector**.

Matching with Features

Problem 2: For each point, how to **match** its **corresponding point** in the other image

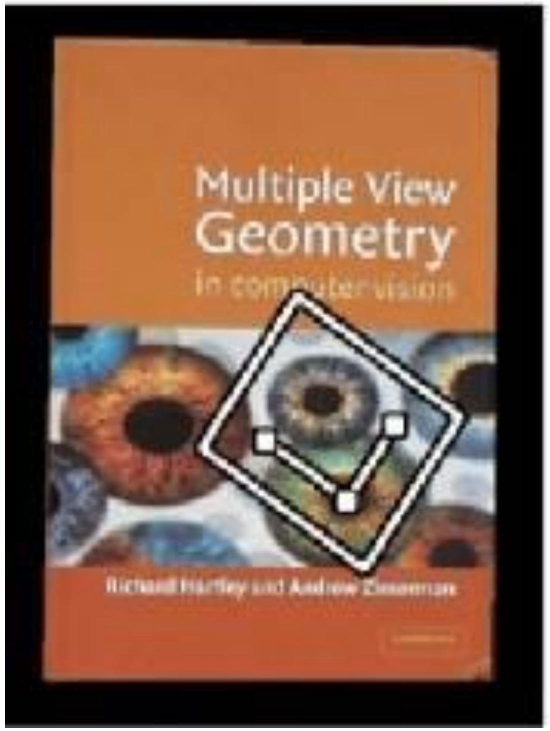


We need a **distinctive** feature descriptor. A descriptor is a "description" of the pixel information around a feature (e.g., patch intensity values, gradient values, etc.). Distinctive means that the descriptor uniquely identifies a feature from other features without ambiguity. This property is called **Distinctiveness** of a feature **descriptor**.

The descriptor must also be **robust to geometric and photometric** changes.

Geometric changes

- **Rotation**
- **Scale** (i.e., zoom)
- **Viewpoint** (i.e., perspective changes)



Photometric Changes (i.e., Illumination changes)

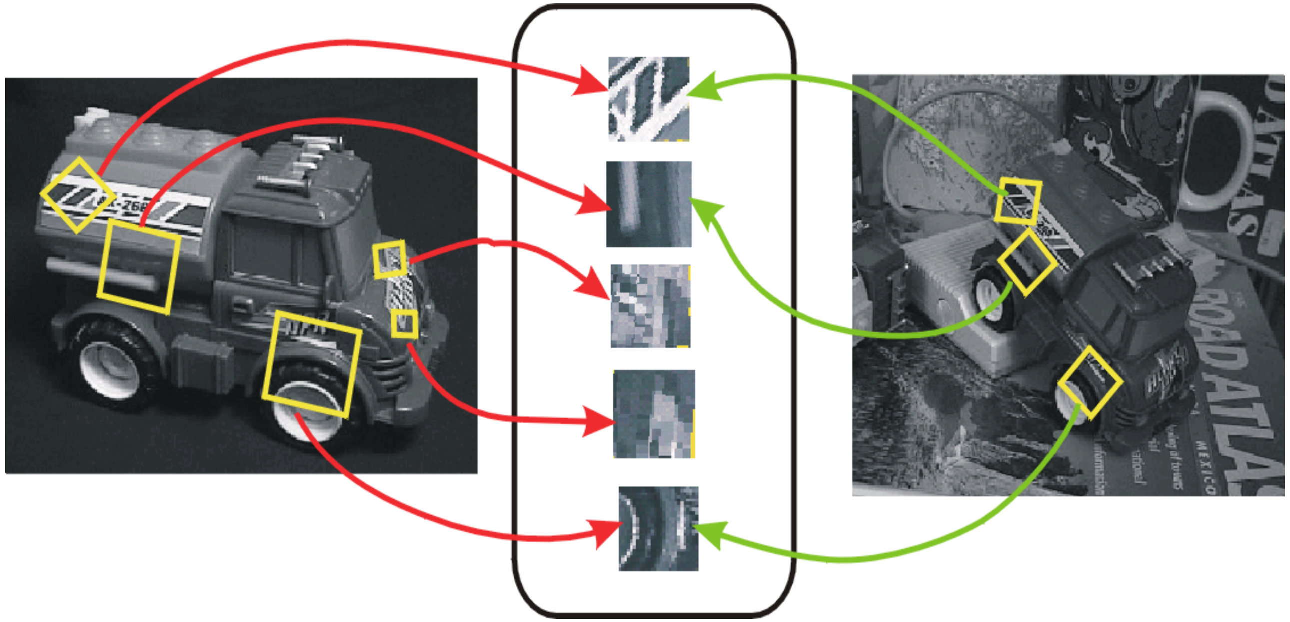
- **Small illumination changes** are modelled with an **affine transformation** (so called *affine illumination changes)*:

$$I'(x,y) = \alpha I(x,y) + \beta$$

Local Invariant Features

The key to feature detection and matching is to find **repeatable features** and **distinctive descriptors** that are **invariant** to geometric and photometric transformations. Basic steps:

1. Detect repeatable and distinctive interest points
2. Extract invariant descriptors

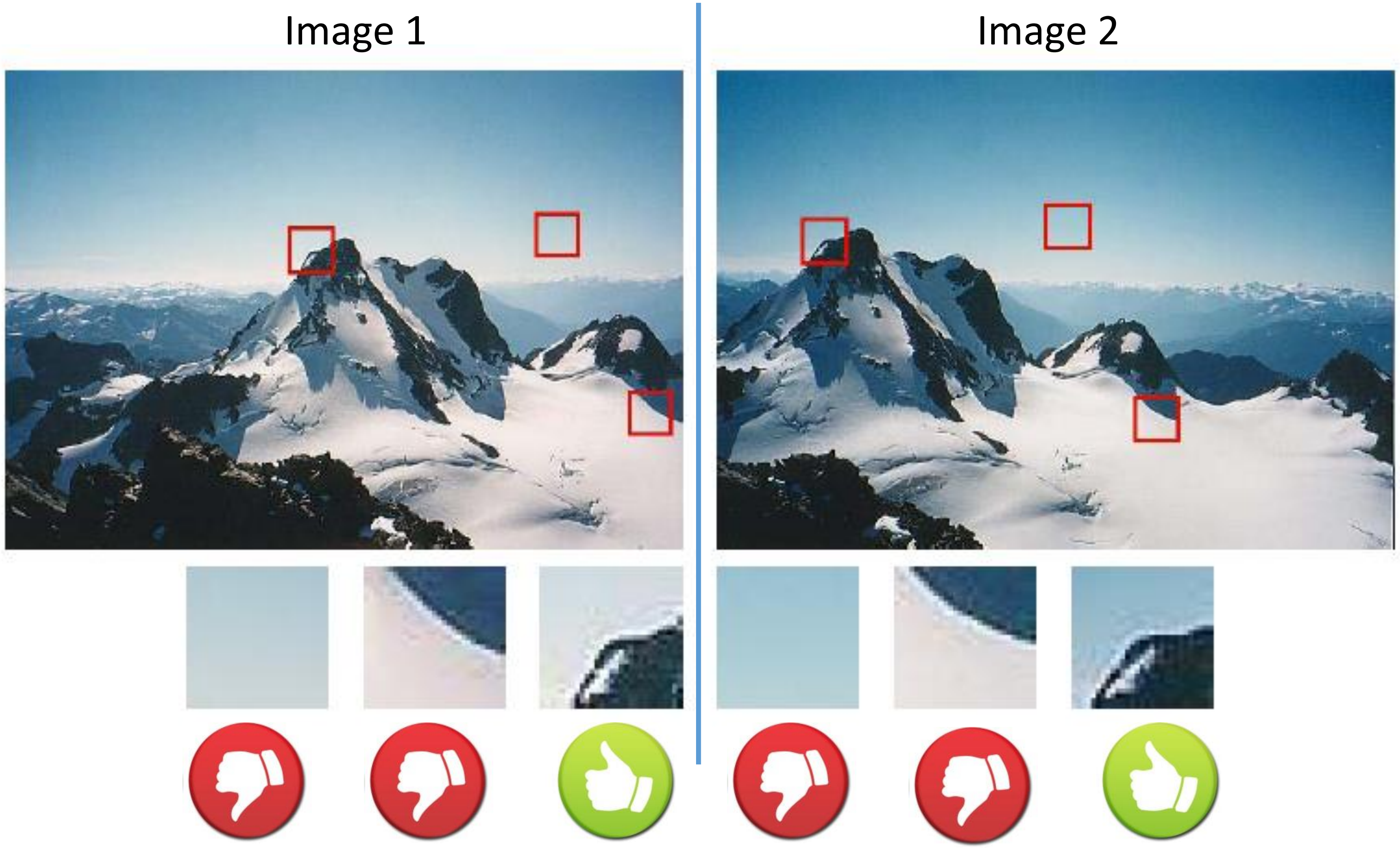


Main questions

- What features are *repeatable* and *distinctive?*
- How to *describe* a feature?
- How to establish *correspondences*, i.e., compute matches?

What is a Repeatable & Distinctive feature?

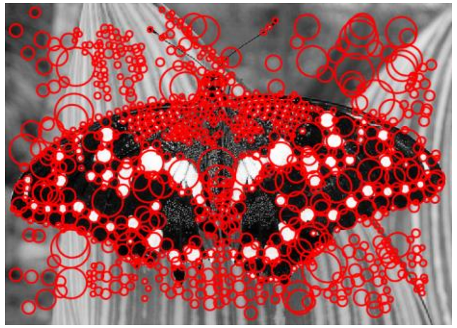
Consider the images below with some patches. Notice how some patches can be localized or matched with higher accuracy than others



Point Features: Corners vs Blob detectors

- A **corner** is defined as the intersection of two or more edges
  - Corners have **high localization** accuracy → corners are good for VO
  - Corners are **less distinctive than blobs**
  - E.g., *Harris, Shi-Tomasi, SUSAN, FAST*

- A **blob** is any other image pattern **that is not a corner** and differs significantly from its neighbors (e.g., a connected region of pixels with similar color, a circle, etc.)
  - **Blobs have less localization accuracy than corners**
  - Blobs are **more distinctive than corners** → blobs **are better for place recognition**
  - E.g., *MSER, LOG, DOG (SIFT), SURF, CenSurE, etc.*

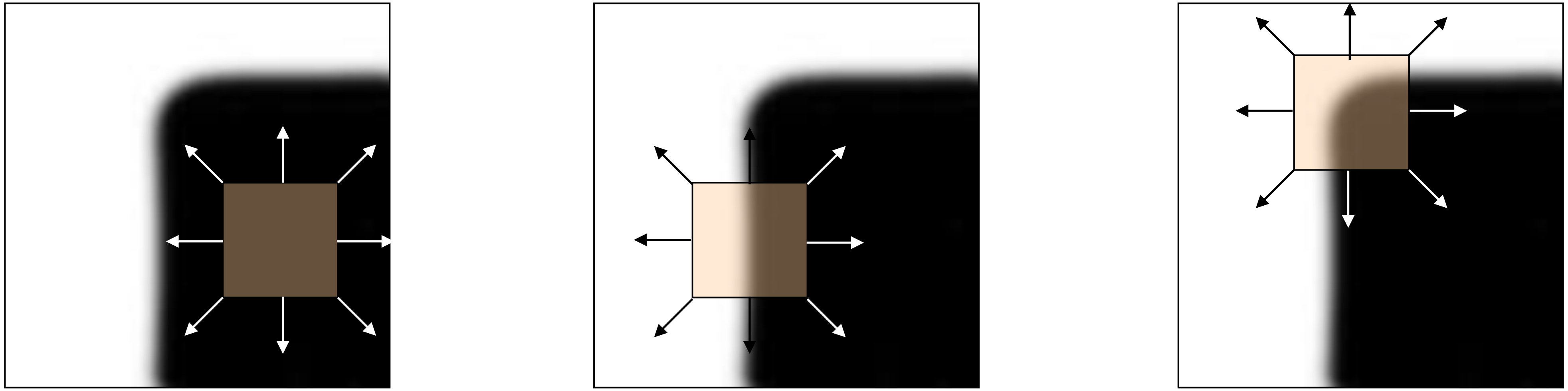


Corner Detection

- Key observation: in the region around a corner, the image gradient has **two or more** dominant directions
- Corners are **repeatable** and **distinctive**

The Moravec Corner detector (1980)

- How do we identify corners? Look at a region of pixels through a small window
- Shifting a window in **any direction** should cause **large intensity changes** (e.g., in SSD)



"flat" region:
no intensity change
(i.e., $SSD \approx 0$ in all directions)

"edge":
no change along the edge direction
(i.e., $SSD \approx 0$ along edge but $\gg 0$ in other directions)

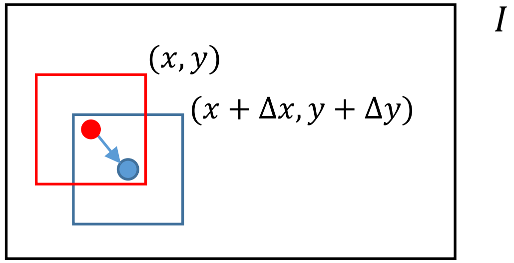
"corner":
significant change in all directions
(i.e., $SSD \gg 0$ in all directions)

H. Moravec, *Obstacle Avoidance and Navigation in the Real World by a Seeing Robot Rover*, PhD thesis, Chapter 5, Stanford University, Computer Science Department, 1980.

The Moravec Corner detector (1980)

Consider the reference patch centered at (x, y) and the shifted window centered at $(x + \Delta x, y + \Delta y)$. The patch has size Ω. The Sum of Squared Differences between them is:

$$SSD(\Delta x, \Delta y) = \sum_{x,y \in \Omega} \left(I(x, y) - I(x + \Delta x, y + \Delta y) \right)^2$$

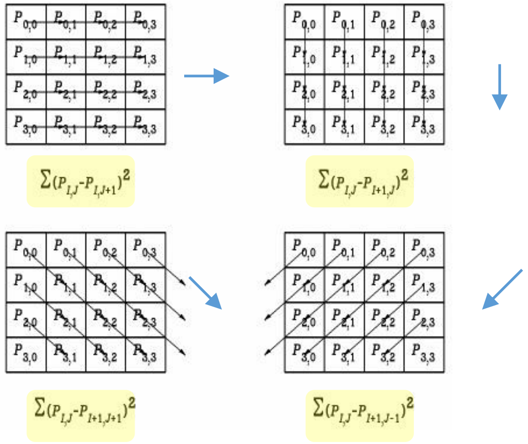


"Sums of squares of differences of pixels adjacent in each of ***four directions (horizontal, vertical and two diagonals)*** *over each window are calculated, and the* ***window's interest measure is the minimum of these four sums****. Features are chosen where the* ***interest measure has local maxima****."* [Moravec'80, PhD thesis, Chapter 5, link]

The disadvantage of the Moravec corner detector is that we need to compute four SSDs, one for each shifted version of the patch (1 pixel right, down, down right, and down left).

Can we make it more efficient?

Can we do it without shifting the patch at all?



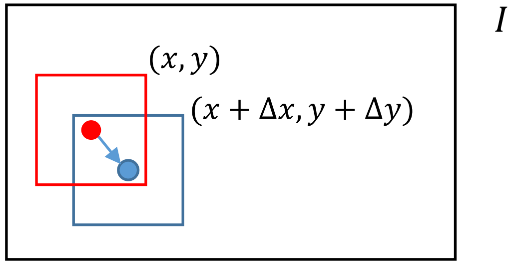
The Harris Corner detector (1988)

It implements the Moravec corner detector without having to physically shift the window but rather by just looking at the patch itself, by using differential calculus.

C.Harris and M.Stephens. "A Combined Corner and Edge Detector". *Proceedings of the 4th Alvey Vision Conference*. 1988. [PDF](PDF).

How do we implement this?

- Consider the reference patch centered at (x, y) and the shifted window centered at $(x + \Delta x, y + \Delta y)$. The patch has size Ω. The Sum of Squared Differences between them is:

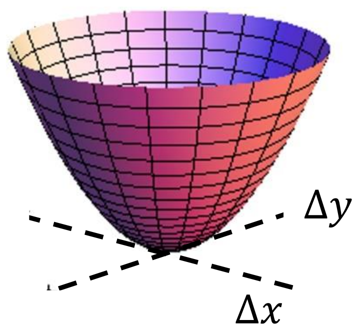


$$SSD(\Delta x, \Delta y) = \sum_{x,y\in\Omega} \left(I(x, y) - I(x + \Delta x, y + \Delta y)\right)^2$$

- Let $I_x = \frac{\partial I(x,y)}{\partial x}$ and $I_y = \frac{\partial I(x,y)}{\partial y}$. Approximating with a 1st order Taylor expansion:

$$I(x + \Delta x, y + \Delta y) \approx I(x, y) + I_x(x, y)\Delta x + I_y(x, y)\Delta y$$

$$\Rightarrow \quad SSD(\Delta x, \Delta y) \approx \sum_{x,y\in\Omega} \left(I_x(x, y)\Delta x + I_y(x, y)\Delta y\right)^2$$



- This is a **quadratic function** in two variables $(\Delta x, \Delta y)$ (i.e., a paraboloid).

How can the shape of this paraboloid reveal whether the patch is a corner, an edge or a constant region?

How do we implement this?

$$SSD(\Delta x, \Delta y) \approx \sum_{x,y\in\Omega} \left(I_x(x,y)\Delta x + I_y(x,y)\Delta y\right)^2$$

Notice that these are NOT matrix products but **pixel-wise** products!

- This can be written in a matrix form as:

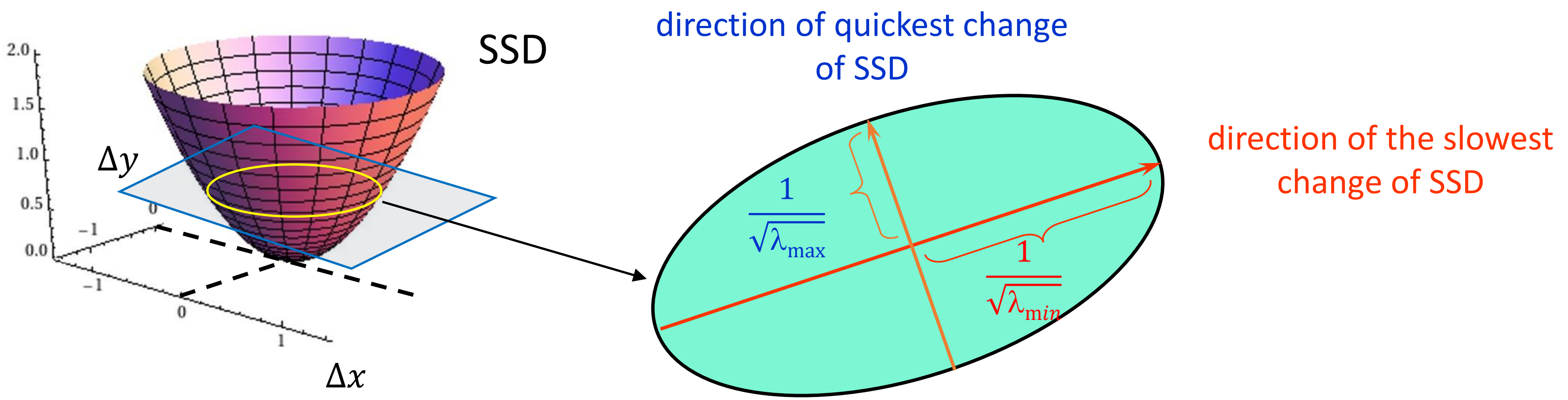
$$SSD(\Delta x, \Delta y) \approx \sum_{x,y\in\Omega} [\Delta x \quad \Delta y] \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = [\Delta x \quad \Delta y] \underbrace{\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}}_{\substack{\text{M} \\ 2^{nd}\text{ moment matrix}}} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

What does this matrix reveal?

- Since M is symmetric, it can always be decomposed into $M = R^{-1}\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$

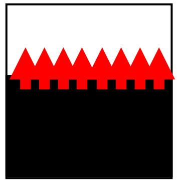
- We can visualize $[\Delta x \quad \Delta y] M \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = const$ as an ellipse with **axes' lengths** determined by the ***eigenvalues*** and the **orientation** determined by R (i.e., the **eigenvectors** of M)

- The two **eigenvectors** identify the **directions of quickest and slowest changes of SSD**



Example

- First, consider an **edge** and a **flat region**
- In presence of noise, we can conclude that if one **eigenvalue** is **much larger than** the other then we have an **edge**. If they are **both small**, then we have a **flat region**.



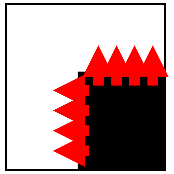
Edge

$$M = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

Flat region

$$M = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

- Now, let's consider an axis-aligned corner:



Corner

$$M = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} = \begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} \cos\frac{\pi}{4} & \sin\frac{\pi}{4} \\ -\sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix}$$

- We can observe that the directions of quickest and slowest change of SSD are at 45 degrees with the x and y axes
- We can thus conclude that if **both eigenvalues are much larger than 0** then we have a **corner**

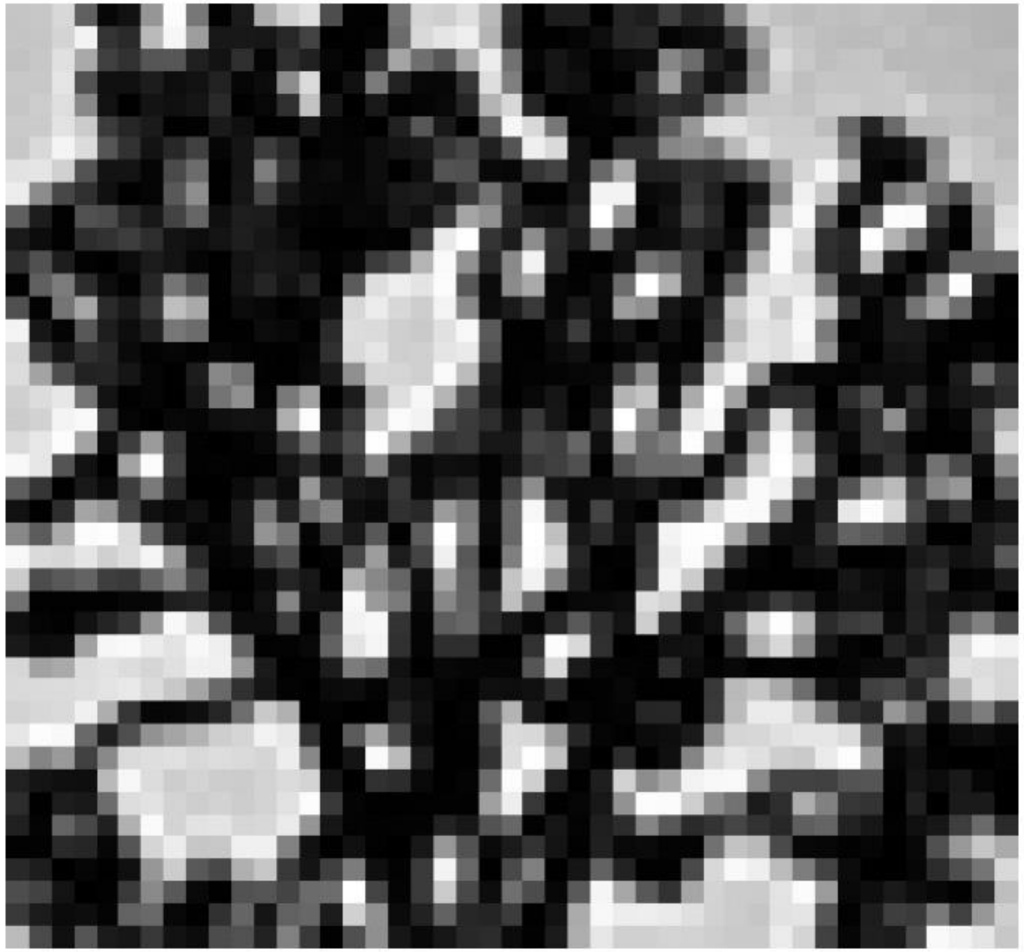
Review: How to compute $\lambda_1, \lambda_2, \mathrm{R}$ from M Eigenvalue/eigenvector

- You can easily prove that λ_1, λ_2 are the **eigenvalues** of M.
- The **eigenvectors** and **eigenvalues** of a square matrix **A** are the vectors **x** and scalars λ that satisfy:

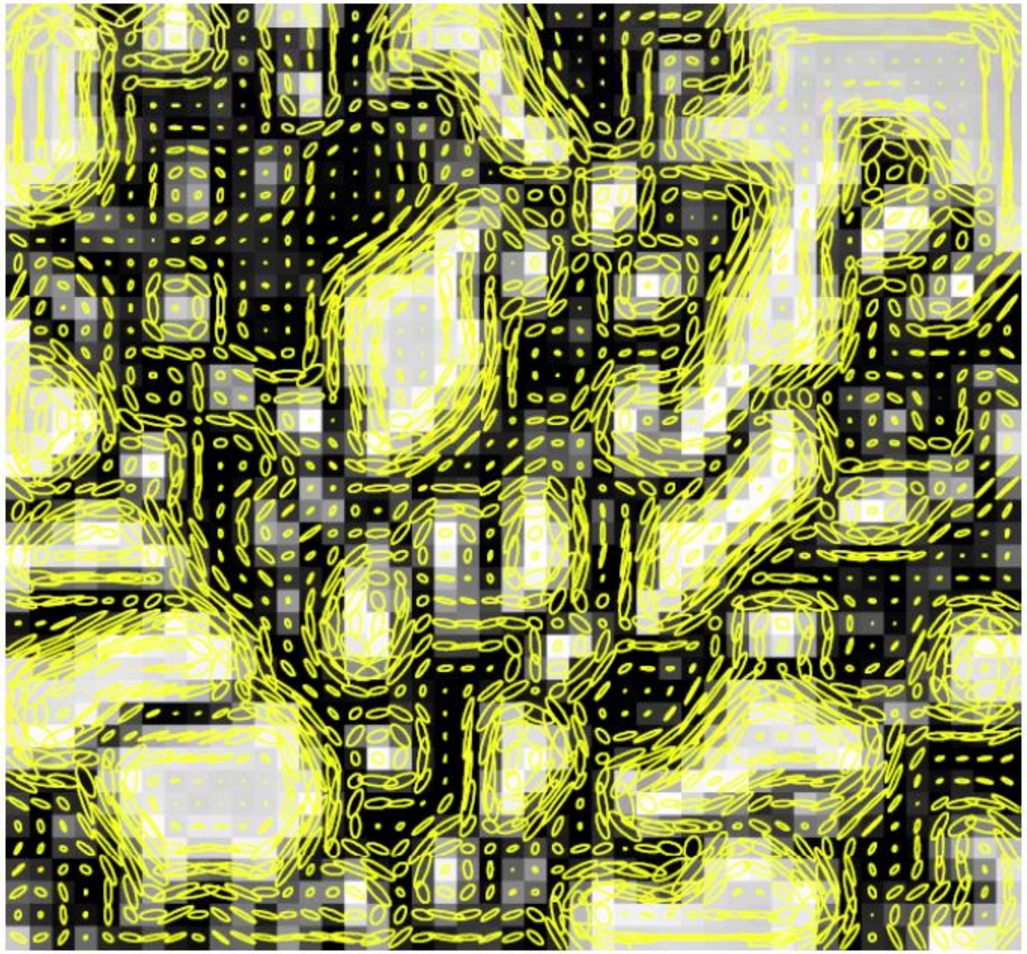
$$Ax = \lambda x$$

- The scalar λ is the **eigenvalue** corresponding to **x**
  - The eigenvalues are found by solving: $$\det(A - \lambda I) = 0$$
  - In our case, $\boldsymbol{A} = \boldsymbol{M}$ is a 2x2 matrix, so we have: $$\det\begin{bmatrix} m_{11} - \lambda & m_{12} \\ m_{21} & m_{22} - \lambda \end{bmatrix} = 0$$
  - The solution is: $$\lambda_{1,2} = \frac{1}{2}\left[(m_{11} + m_{22}) \pm \sqrt{4m_{12}m_{21} + (m_{11} - m_{22})^2}\right]$$
  - Once you know λ, you find the two eigenvectors $\boldsymbol{x}$ (i.e., the two columns of R) by solving: $$\begin{bmatrix} m_{11} - \lambda & m_{12} \\ m_{21} & m_{22} - \lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$$

Visualization of 2nd moment matrices



Visualization of 2^{nd} moment matrices



NB: here the ellipses here are plotted proportionally to the eigenvalues and not as iso-SSD ellipses as explained before. So small ellipses here denote a flat region, and big ones, a corner.

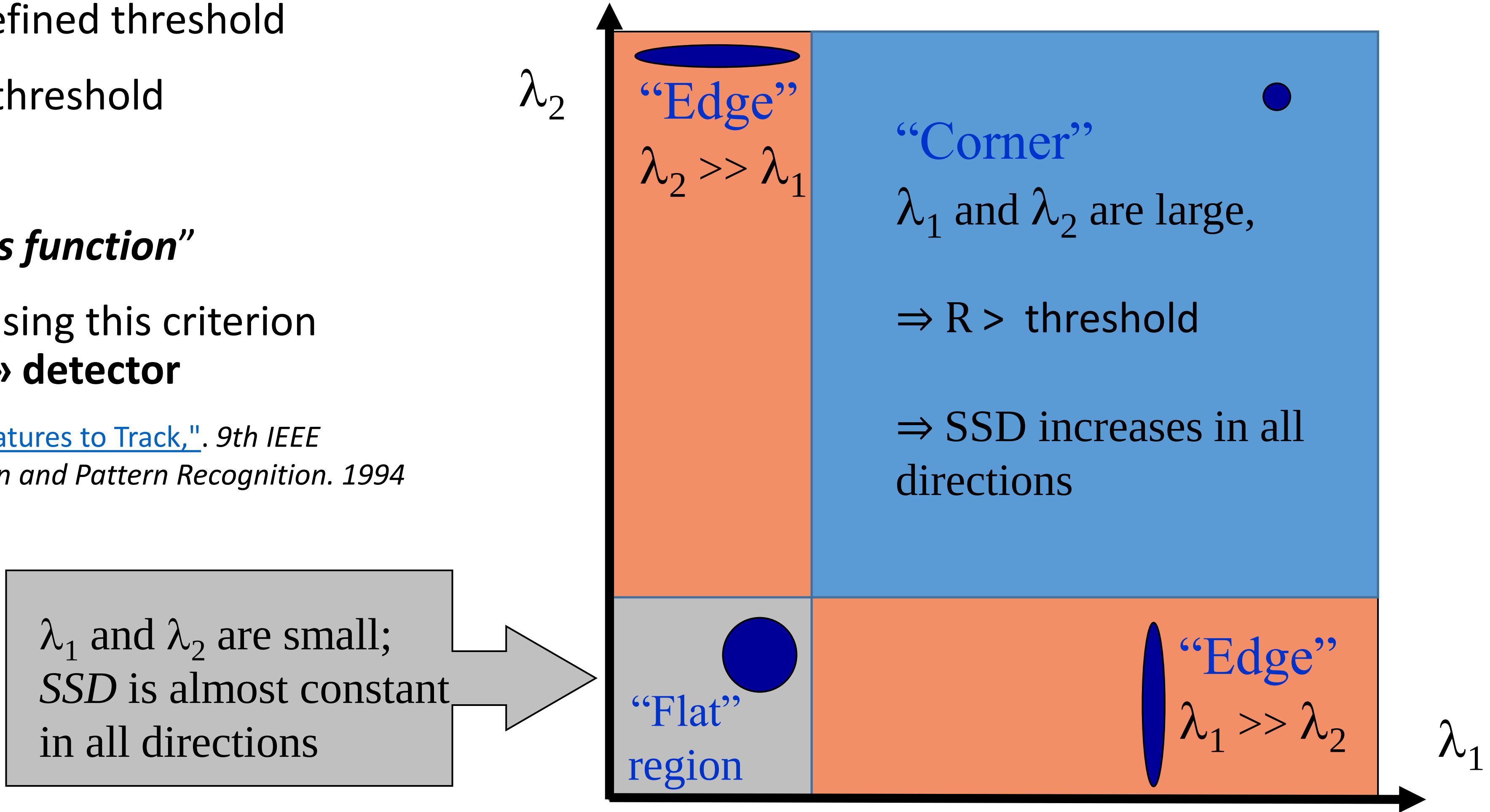
Interpreting the eigenvalues

- Classification of image points using eigenvalues of *M*
- A corner can then be identified by checking whether the minimum of the two eigenvalues of M is larger than a certain user-defined threshold

  $\Rightarrow R = \min(\lambda_1, \lambda_2) >$ threshold

- R is called "***cornerness function***"
- The corner detector using this criterion **is called «Shi-Tomasi» detector**

  J. Shi and C. Tomasi. "Good Features to Track,". *9th IEEE Conference on Computer Vision and Pattern Recognition. 1994*



Interpreting the eigenvalues

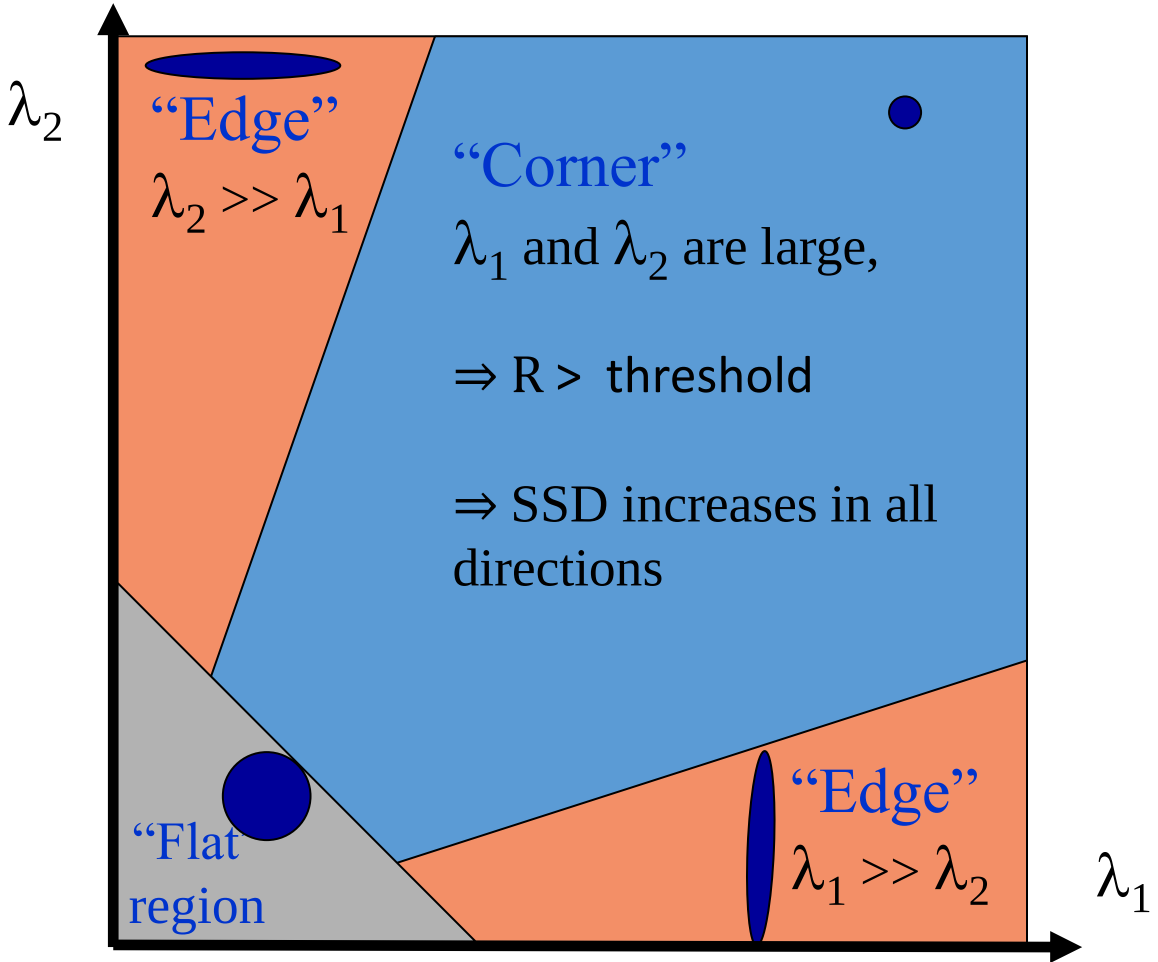
- Computation of λ_1 and λ_2 is expensive ➪ Harris & Stephens suggested using a **different cornerness function**:

$$R = \lambda_1 \lambda_2 - k(\lambda_1 + \lambda_2)^2 = \det(M) - k \operatorname{trace}^2(M)$$

k is a *magic number* in the range (0.04 to 0.15)

- The corner detector using this criterion **is called «Harris» detector**

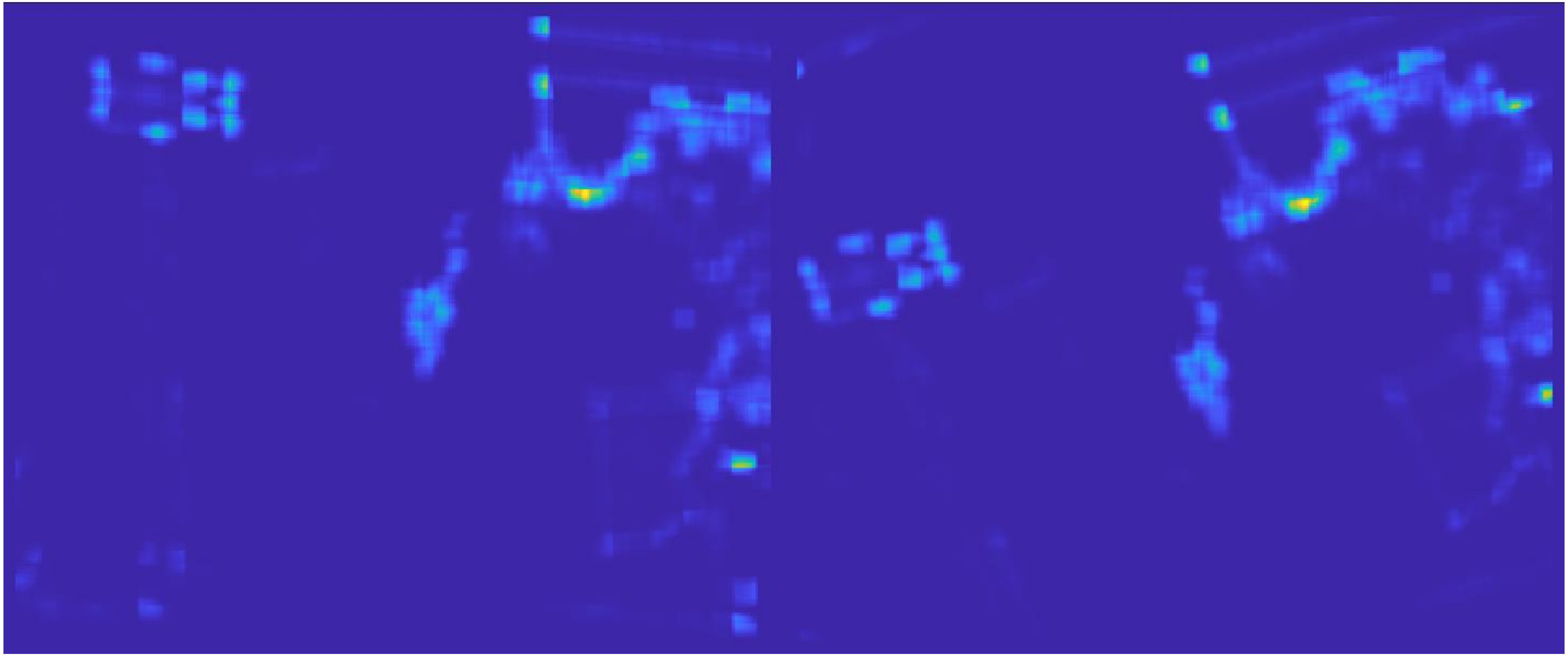
C.Harris and M.Stephens. "A Combined Corner and Edge Detector“ , *Proceedings of the 4th Alvey Vision Conference*, 1988.



Harris Detector: Workflow

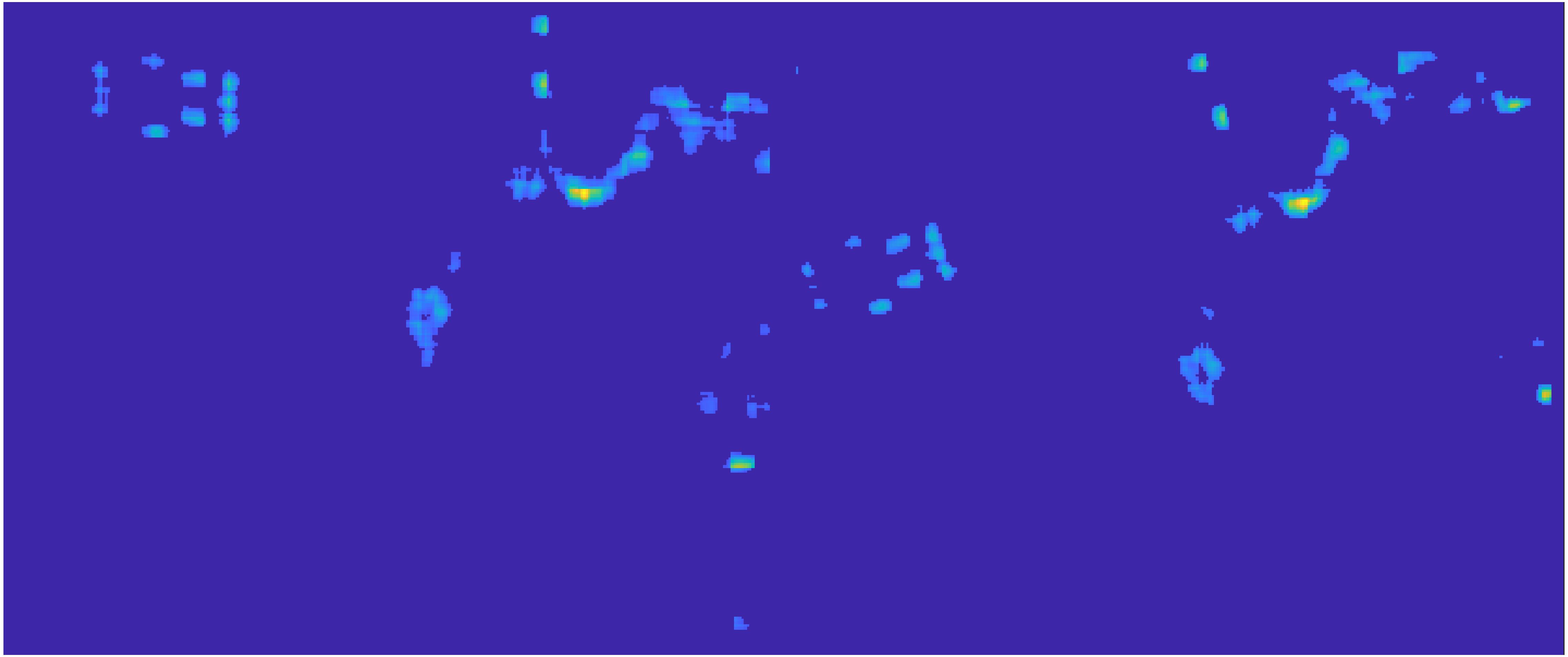
Harris Detector: Workflow

- Compute corner response R



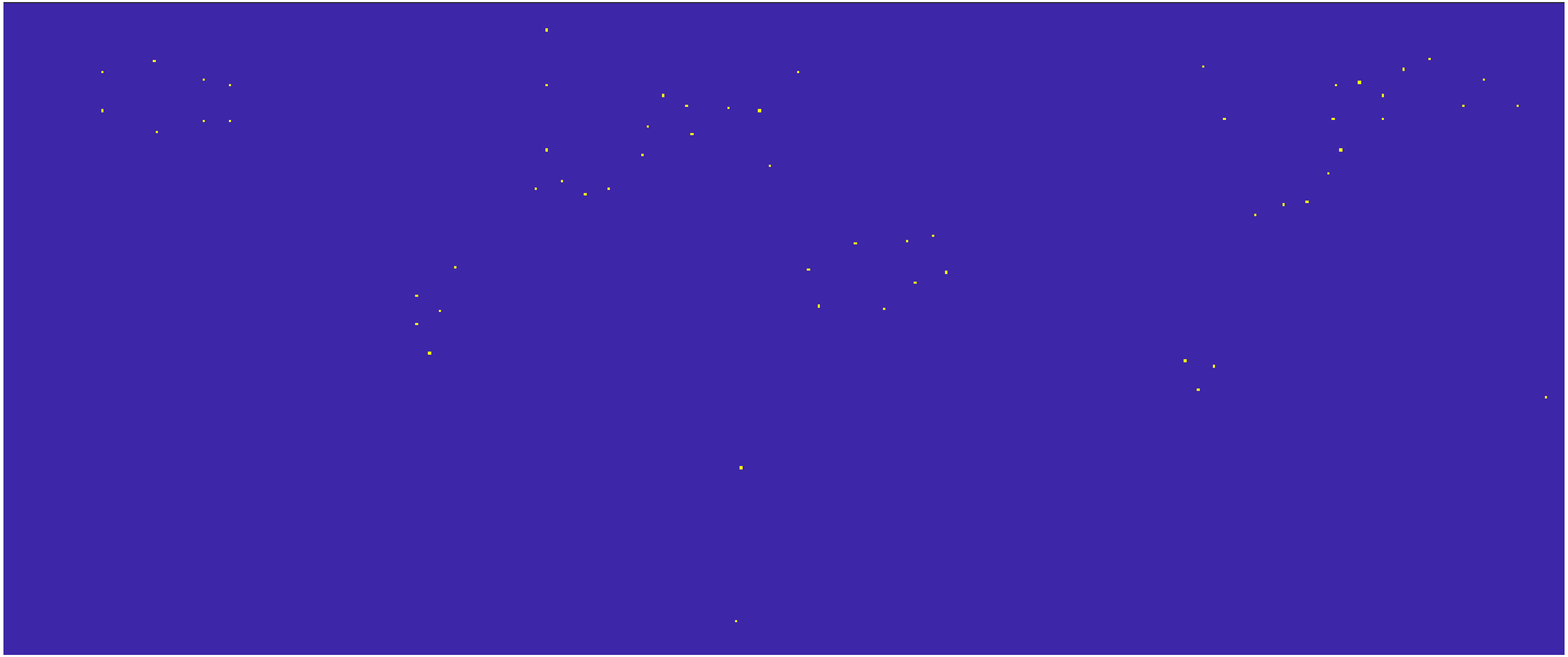
Harris Detector: Workflow

- **Thresholding**: Find points with large corner response: $R > threshold$



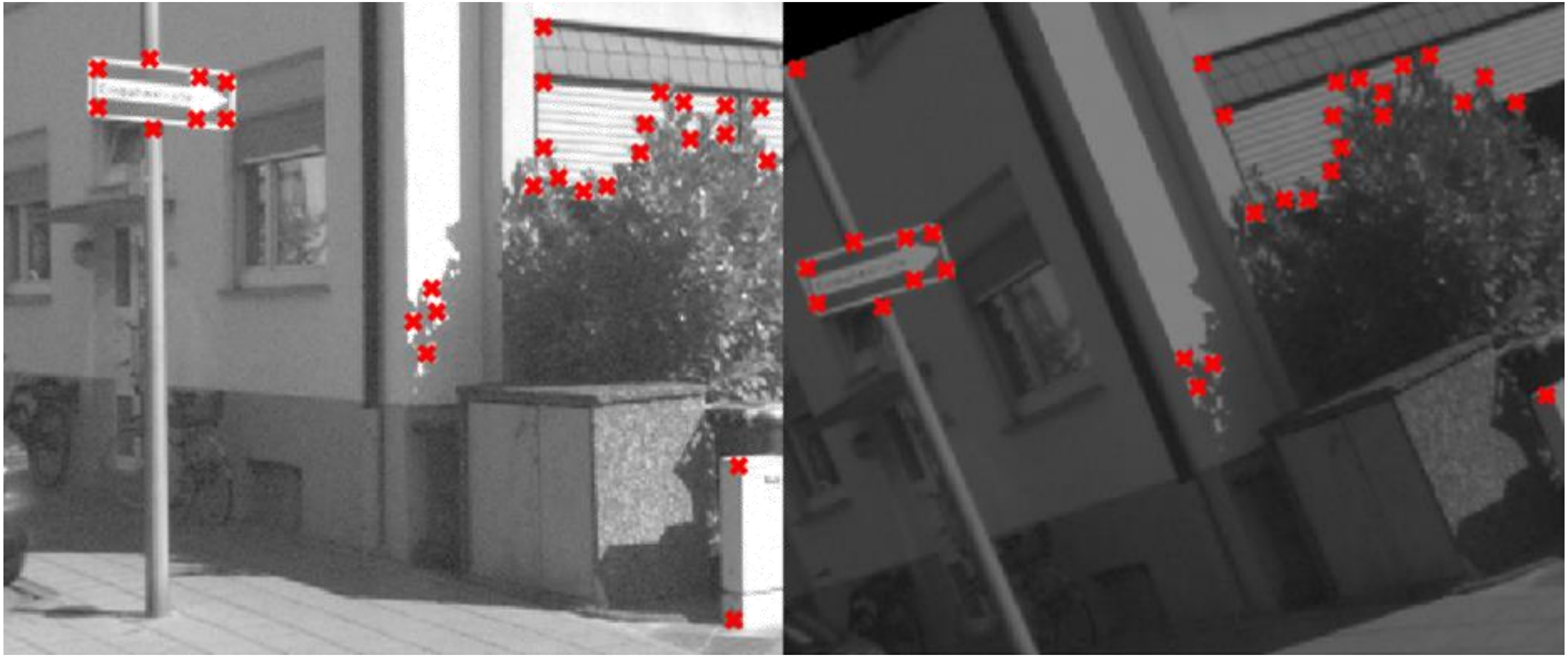
Harris Detector: Workflow

- **Non-Maxima Suppression**: detect local maxima of thresholded R



Harris Detector: Workflow

What parameters can we tune to detect more or fewer corners?



Harris (or Shi-Tomasi) Corner Detector Algorithm

Algorithm:

1. Compute derivatives in *x* and *y* directions (I_x, I_y) e.g. with *Sobel filter*
2. Compute ${I_x}^2, {I_y}^2, I_x I_y$
3. Convolve ${I_x}^2, {I_y}^2, I_x I_y$ with a *box filter* to get $\sum {I_x}^2, \sum {I_y}^2, \sum I_x I_y$, which are the entries of the matrix M (optionally use a Gaussian filter instead of a box filter to avoid aliasing and give more "weight" to the central pixels)
4. Compute Corner Measure R according to Shi-Tomasi or Harris
5. Find points with large corner response $(R > \text{threshold})$
6. Take the points of local maxima of *R*

From now on, whenever we talk about the Harris corner detector we will be referring to either the original Harris detector (1988) or to its modification by Shi-Tomasi (1994).
The Shi-Tomasi detector, despite being a bit more expensive, yet has a small advantage… see next slide

Harris vs. Shi-Tomasi

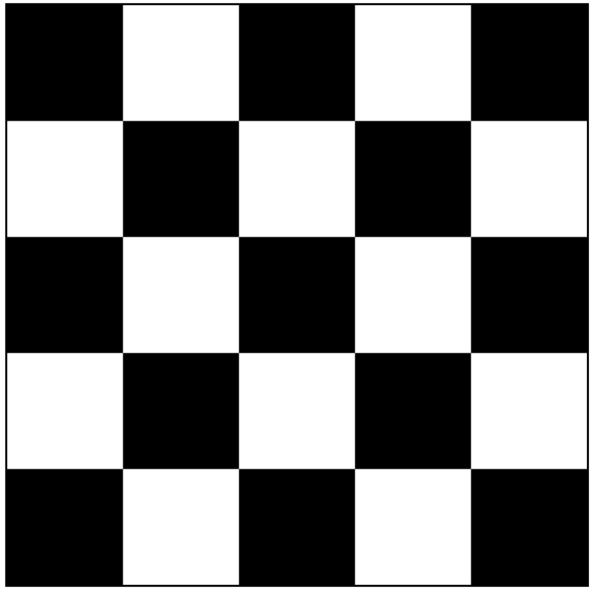
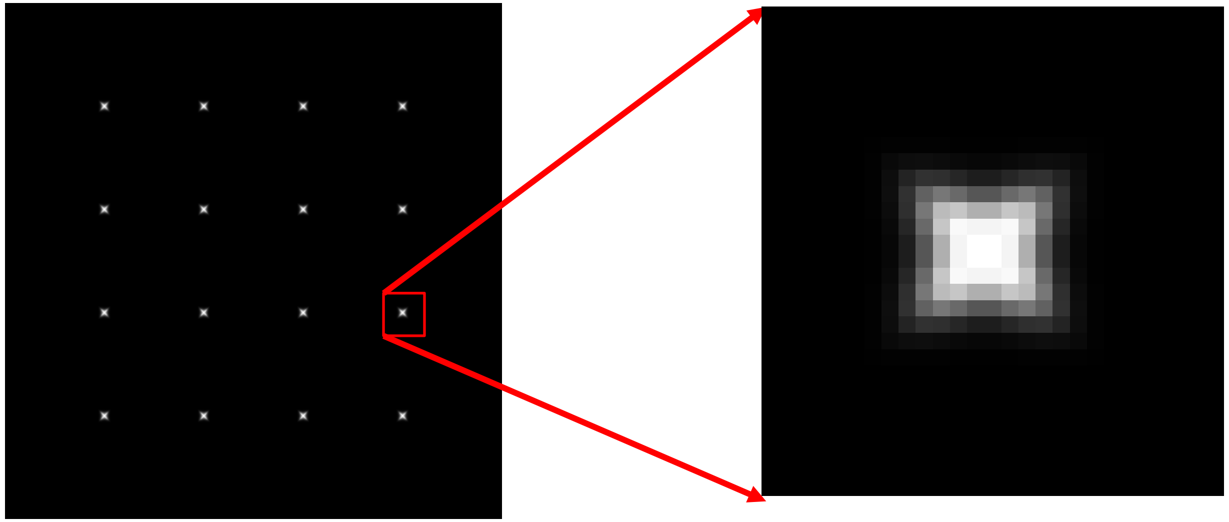
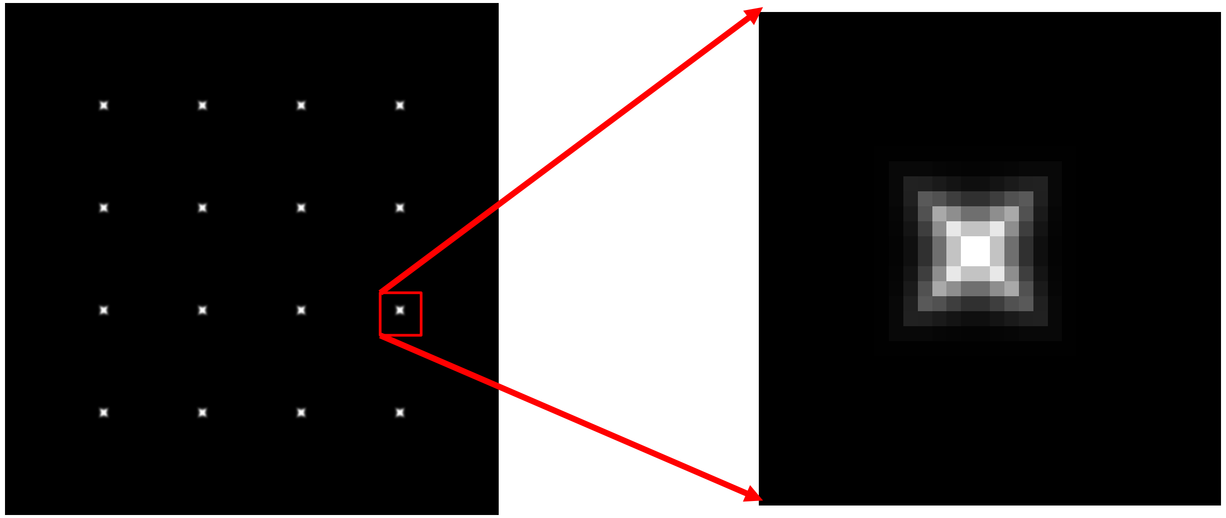


Image I



Harris'cornerness response



Shi-Tomasi's cornerness response

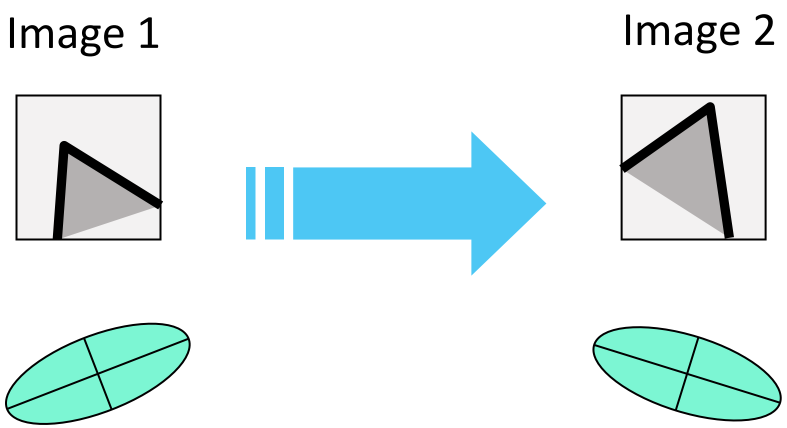
Harris Detector: Some Properties

Repeatability:

- How does the Harris detector behave with **geometric and photometric changes**, i.e. can it re-detect the same corners when the image exhibits changes in
  - **Rotation**,
  - **Scale** (zoom),
  - **View-point**,
  - **Illumination** ?

Harris Detector: Some Properties

- The Harris detector is **rotation invariant**

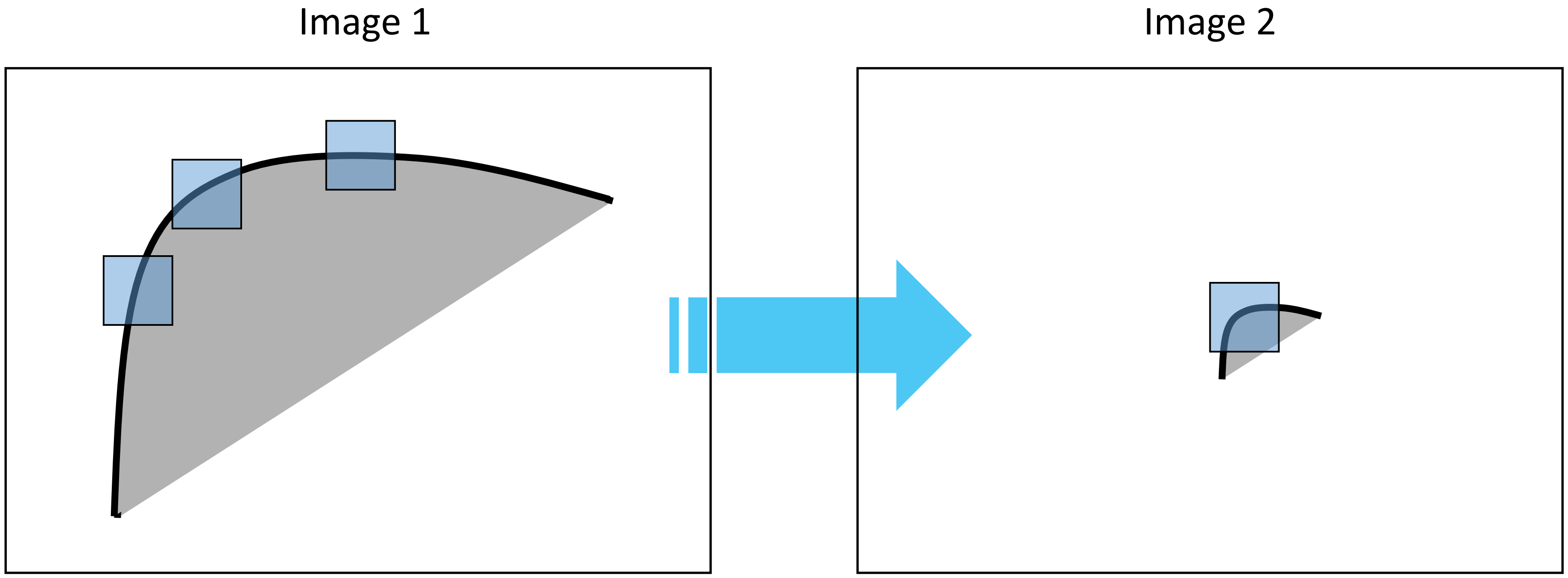


Ellipse rotates but its shape (i.e., eigenvalues of M) remains the same

Corner response R is **invariant to image rotation**

Harris Detector: Some Properties

- The Harris detector is **not scale invariant**



All points will be classified as **edges**

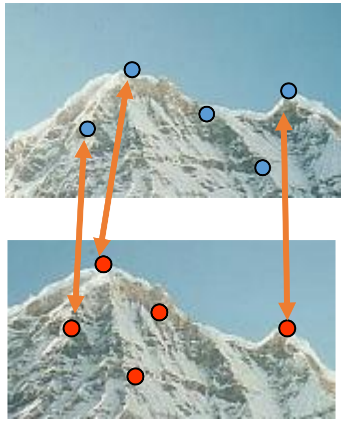
Corner!

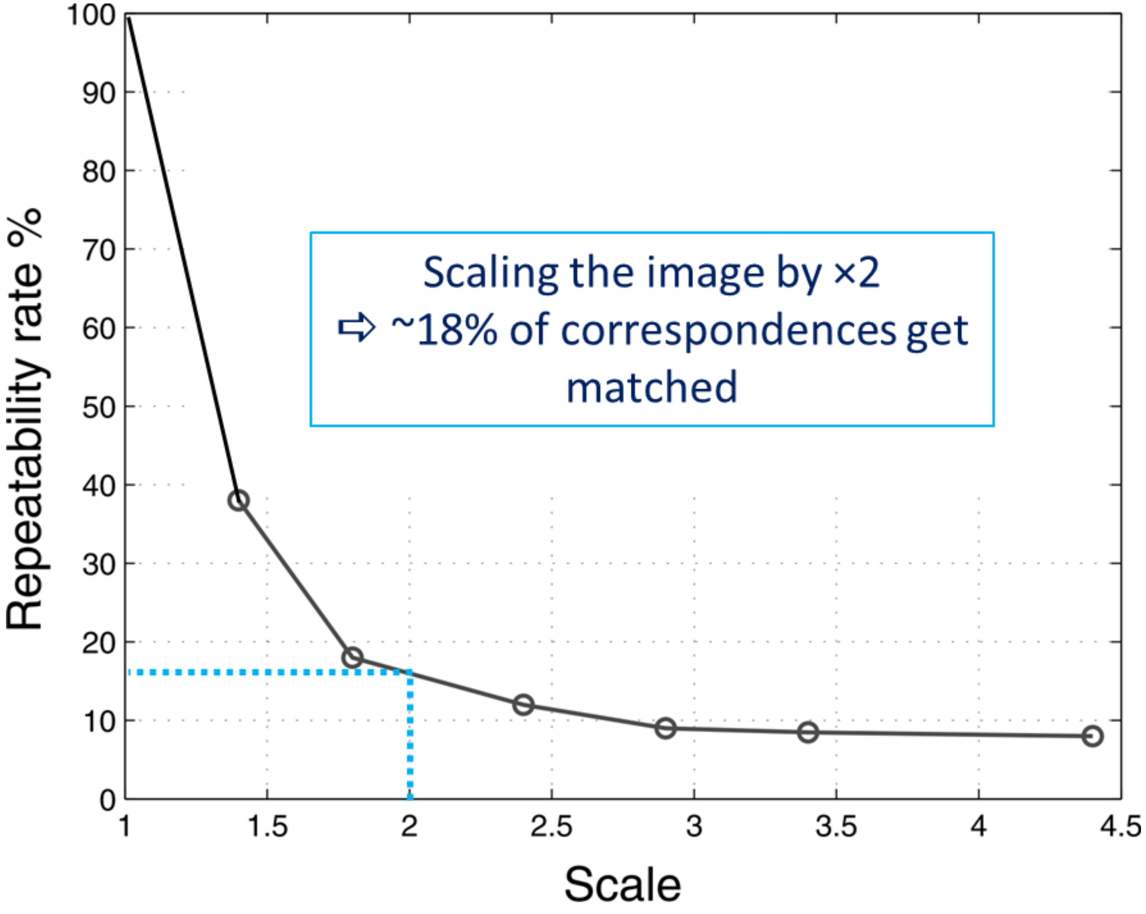
Harris Detector: Some Properties

- **Repeatability** of the Harris detector for different scale changes

Repeatability=

$$\frac{\text{\# correspondences detected}}{\text{\# correspondences present}}$$





Harris Detector: Some Properties

- Is it invariant to:
  - Affine illumination changes?
    - yes, why?
  - Any monotonic, nonlinear illumination changes?
    - yes, why?
    - **Hint**: remember that Harris corners are local maxima of the cornerness response function
  - View point invariance?
    - Does the same corner look like a corner from a different view point?
      - It depends on the view point change, why?
        Hint: remember that Harris corners are local maxima of the cornerness response function

Summary (things to remember)

- Filters as templates
- Correlation as a scalar product
- Similarity metrics: NCC (ZNCC), SSD (ZSSD), SAD (ZSAD), Census Transform
- Point feature detection
    - Properties and invariance to transformations
        - Challenges: rotation, scale, view-point, and illumination changes
    - Extraction
        - Moravec
        - Harris and Shi-Tomasi
            - Invariance to rotation, scale, illumination changes

Readings

- Ch. 7.1 and Ch. 9.1 of Szeliski book, 2nd Edition
- Chapter 4 of Autonomous Mobile Robots book: link
- Ch. 13.3 of Peter Corke book

Understanding Check

Are you able to:

- Explain what is template matching and how it is implemented?
- Explain what are the limitations of template matching? Can you use it to recognize cars?
- Illustrate the similarity measures: SSD, SAD, NCC, and Census transform?
- What is the intuitive explanation behind SSD and NCC?
- Explain what are good features to track? In particular, can you explain what are corners and blobs together with their pros and cons? How is their localization accuracy?
- Explain the Harris corner detector? In particular:
  - Use the Moravec definition of corner, edge and flat region.
  - Show how to get the second moment matrix from the definition of SSD and first order approximation (show that this is a quadratic expression) and what is the intrinsic interpretation of the second moment matrix using a paraboloid and using an ellipse?
  - What is the M matrix like for an edge, for a flat region, for an axis-aligned (90-degree) corner and for a non-axis aligned corner?
  - What do the eigenvalues of M reveal?
  - Can you compare Harris detection with Shi-Tomasi detection?
  - Can you explain whether the Harris detector is invariant to illumination or scale changes? Is it invariant to view point changes?
  - What is the repeatability of the Harris detector after rescaling by a factor of 2?