Vision Algorithms for Mobile Robotics

Lecture 04
Image Filtering

Davide Scaramuzza

http://rpg.ifi.uzh.ch

Today's exercise session replaced by lecture

21.09.2023	**Lecture 01 - Introduction to Computer Vision and Visual Odometry** **Exercise: Camera Notation Tutorial**	Scaramuzza Leonard, Jiaxu
28.09.2023	**Lecture 02 - Image Formation: perspective projection and camera models** **Exercise 01- Augmented reality wireframe cube**	Scaramuzza Leonard, Jiaxu
05.10.2023	**Lecture 03 - Camera Calibration** **Exercise 02 - PnP problem**	Leonard Leonard, Jiaxu
12.10.2023	**Lecture 03 continued** **Lecture 04 - Filtering & Edge detection** **Exercise session replaced by continuation of Lecture 4**	Scaramuzza
19.10.2023	**Lecture 05 - Point Feature Detectors, Part 1** **Exercise 03 - Harris detector + descriptor + matching**	Scaramuzza Leonard, Jiaxu
26.10.2023	**Lecture 06 - Point Feature Detectors, Part 2** **Exercise 04 - SIFT detector + descriptor + matching**	Leonard Leonard, Jiaxu
02.11.2023	**Lecture 07 - Multiple-view Geometry 1** **Exercise 05 - Stereo vision: rectification, epipolar matching, disparity, triangulation**	Scaramuzza Leonard, Jiaxu
09.11.2023	**Lecture 08 - Multiple-view Geometry 2** **Exercise 06 - Eight-Point Algorithm**	Scaramuzza Leonard, Jiaxu
16.11.2023	**Lecture 09 - Multiple-view Geometry 3** **Exercise 07 - P3P algorithm and RANSAC**	Scaramuzza Leonard, Jiaxu
23.11.2023	**Lecture 10 - Multiple-view Geometry 4** **Continuation of Lecture 10 + Exercise session on Intermediate VO Integration**	Scaramuzza Leonard, Jiaxu
30.11.2023	**1st hour: seminar by Dr. Jeff Delaune from NASA-JPL: "Vision-Based Navigation for Mars Helicopters."** **2nd hour: Lecture 11 - Optical Flow and KLT Tracking** **Exercise 08 - Lucas-Kanade tracker**	NASA Leonard Leonard, Jiaxu
07.12.2023	**Lecture 12a (1st hour) - Place Recognition** **Lecture 12b (2nd hour) - Dense 3D Reconstruction** **Lecture 12c (3rd and 4th hour, replaces exercise) - Deep Learning Tutorial** **Optional Exercise on Place Recogntion**	Scaramuzza Scaramuzza Leonard
14.12.2023	**Lecture 13 - Visual inertial fusion** **Exercise 09 - Bundle Adjustment**	Scaramuzza Leonard, Jiaxu
21.12.2023	**Lecture 14 - Event-based vision + lab visit after the lecture** **Exercise session: Final VO Integration**	Scaramuzza Leonard, Jiaxu

Today's Outline

- Low-pass filtering
  - Linear filters
  - Non-linear filters
- Edge Detection
  - Canny edge detector

Image filtering

- The word *filter* comes from frequency-domain processing, where “filtering” refers to the process of accepting or rejecting certain frequency components
- We distinguish between low-pass and high-pass filtering
  - A **low-pass filter** smooths an image (retains low-frequency components)
  - A **high-pass filter** retains the contours (also called edges) of an image (high frequency)

Low-pass filtered image

High-pass filtered image

Today’s Outline

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Low-pass filtering applied to noise reduction

- **Salt and pepper noise**: random occurrences of black and white pixels
- **Impulse noise:** random occurrences of white pixels
- **Gaussian noise**: variations in intensity drawn from a Gaussian distribution

Salt and pepper noise and Impulse noise are caused by

- data transmission errors,
- failure in memory cell, or
- analog-to-digital converter errors.

Original

Salt and pepper noise

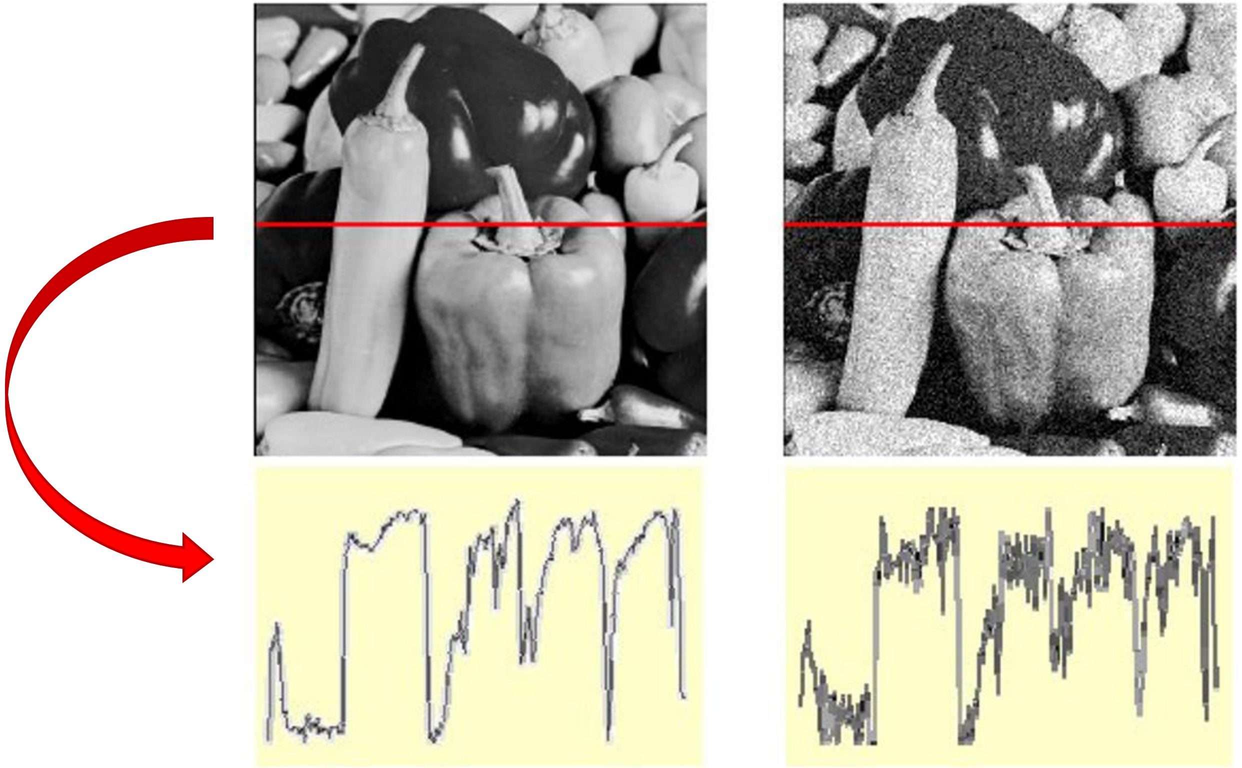
Impulse noise

Gaussian noise

Additive Independent and Identically Distributed Gaussian noise

It is Independent and Identically Distributed (I.I.D.) noise drawn from a zero-mean Gaussian distribution:

$$\eta(x,y) \sim \mathrm{N}(0,\sigma)$$



Ideal image Noise

$$I(x,y) \quad = \quad I'(x,y) \;+\; \eta(x,y)$$

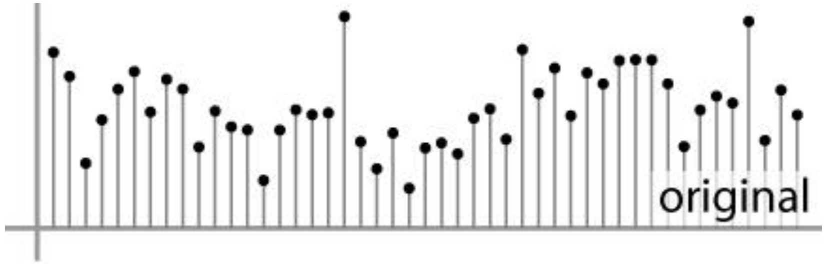
How can we reduce the noise to recover the “ideal image”?

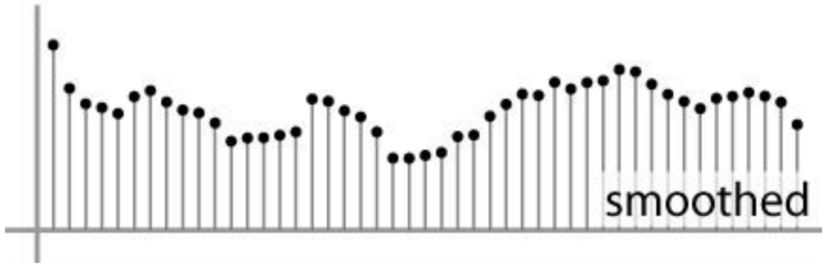
Moving average

- **Replaces each pixel** with an **average** of all the values in its neighborhood
- **Assumptions**:
  - Expect **noise process to be i.i.d. Gausian**
  - Expect **pixels to be like** their **neighbors**

Moving average

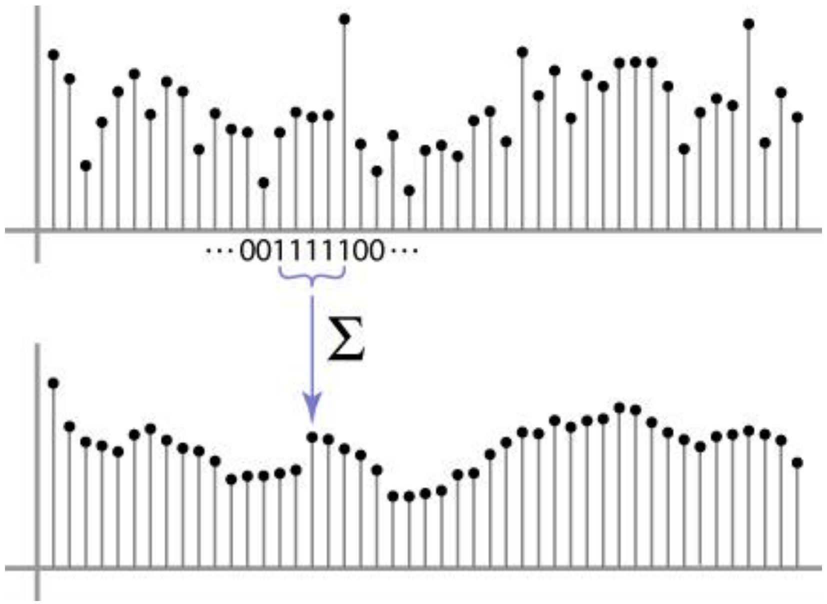
- **Replaces each pixel** with an **average** of all the values in its neighborhood
- Moving average in 1D:





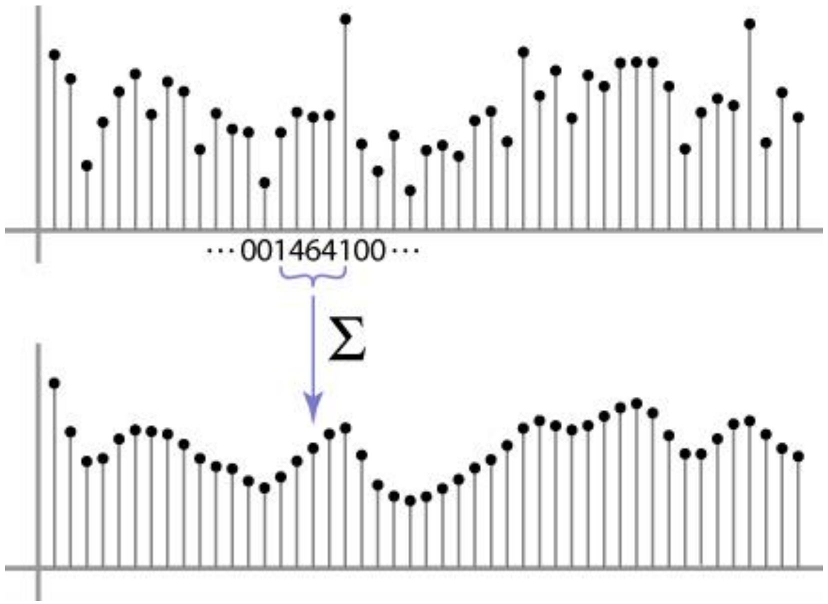
Weighted Moving Average

- Can add weights to our moving average
- **Uniform weights:** [1, 1, 1, 1, 1] / 5



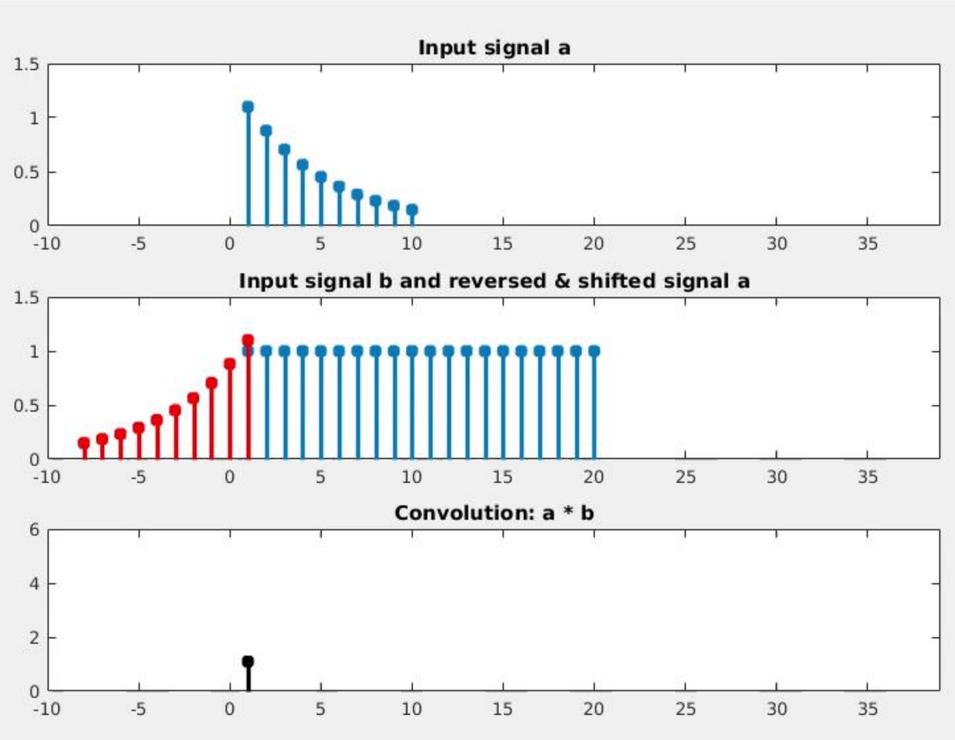
Weighted Moving Average

- **Non-uniform weights:** [1, 4, 6, 4, 1] / 16



This operation is called *convolution*

- Example of convolution between two signals
  - One of the sequences is flipped (right to left) before sliding over the other
  - Notation: $a * b$
  - Nice properties: linearity, associativity, commutativity, etc.



This operation is called *convolution*

- Example of convolution between two signals
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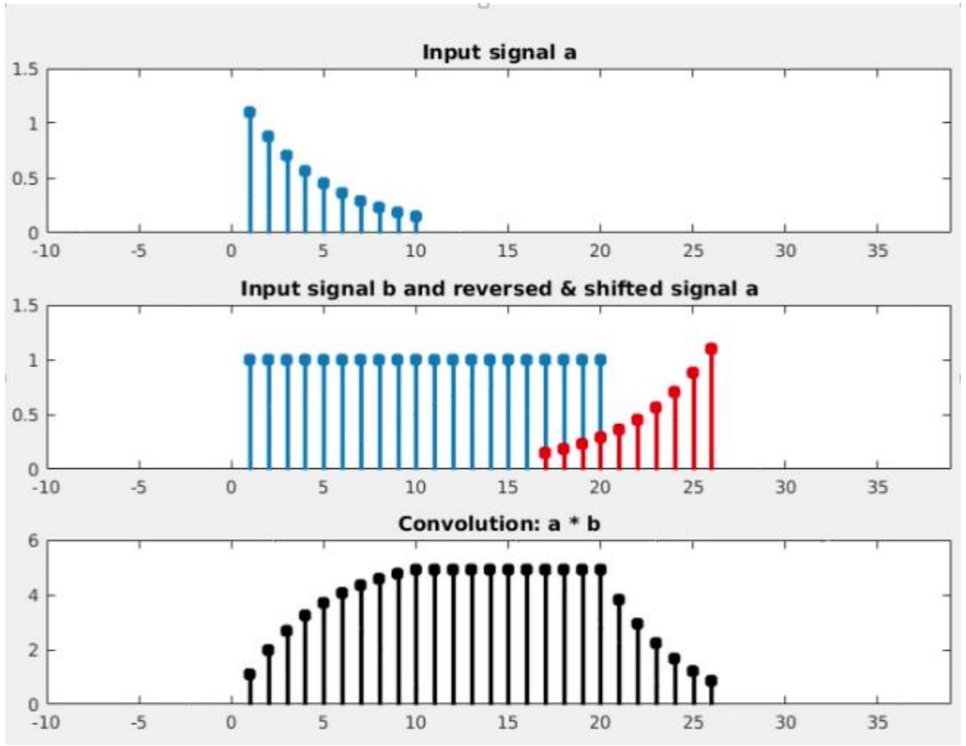
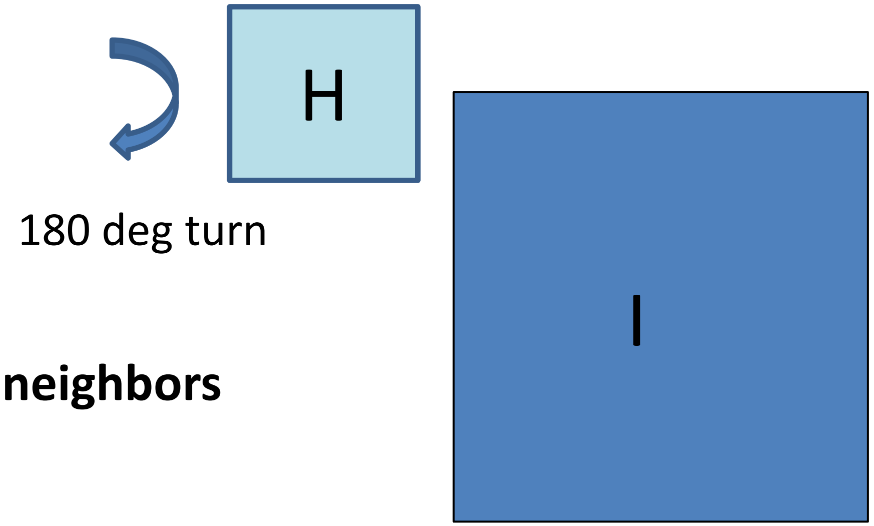


2D Filtering via 2D Convolution

- Flip the filter in both dimensions (bottom to top, right to left) (=180 deg turn)
- Then slide the filter over the image and compute sum of products

$$I'[x,y] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} I[x-u, y-v]H[u,v]$$



$$I' = I * H$$

- Convolution **replaces each pixel with a weighted sum of its neighbors**
- The **filter *H*** is also called "**kernel**" or "**mask**"

Review: Convolution vs. Cross-correlation

Convolution: $I' = I * H$

- Properties: **linearity, associativity, commutativity**

$$I'[x,y] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} I[x-u, y-v]H[u,v]$$

For a Gaussian or box filter, will the output of convolution and correlation be different?

Cross-correlation: $I' = I \otimes H$

Properties: linearity, but **no associativity** and **no commutativity**

$$I'[x,y] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} I[x+u, y+v]H[u,v]$$

Box Filter

Input image

$I[x, y]$

Filtered image

$I'[x, y]$

"box filter"

$\frac{1}{9}$

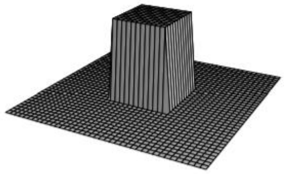
1	1	1
1	1	1
1	1	1

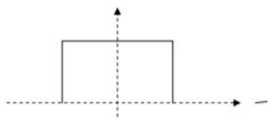
			0	0	0	0	0	0	0
			0	0	0	0	0	0	0
			90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

	0								

"box filter"

$^1/_9$	$^1/_9$	$^1/_9$
$^1/_9$	$^1/_9$	$^1/_9$
$^1/_9$	$^1/_9$	$^1/_9$

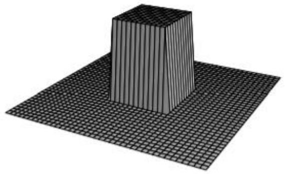


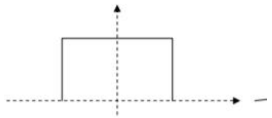


Box Filter

"box filter"

$^1/_9$	$^1/_9$	$^1/_9$
$^1/_9$	$^1/_9$	$^1/_9$
$^1/_9$	$^1/_9$	$^1/_9$





Input image

$I[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Filtered image

$I'[x, y]$

	0	10							

Box Filter

Input image

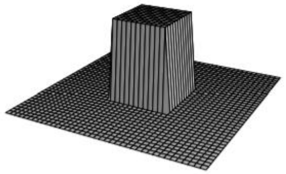
$I[x,y]$

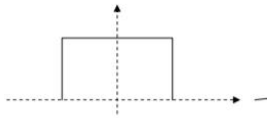
Filtered image

$I'[x,y]$

"box filter"

$^1/_9$	$^1/_9$	$^1/_9$
$^1/_9$	$^1/_9$	$^1/_9$
$^1/_9$	$^1/_9$	$^1/_9$





Input image:

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

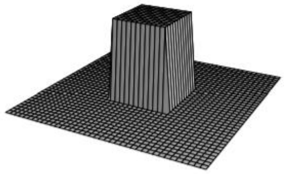
Filtered image:

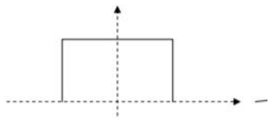
	0	10	20						

Box Filter

“box filter”

$^1/_9$	$^1/_9$	$^1/_9$
$^1/_9$	$^1/_9$	$^1/_9$
$^1/_9$	$^1/_9$	$^1/_9$





Input image

$$I[x, y]$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Filtered image

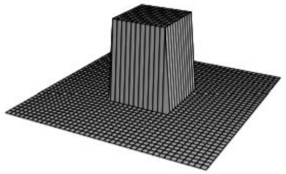
$$I'[x, y]$$

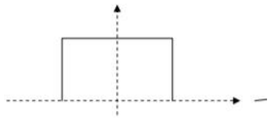
	0	10	20	30					

Box Filter

"box filter"

$$\begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix}$$





Input image

$I[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Filtered image

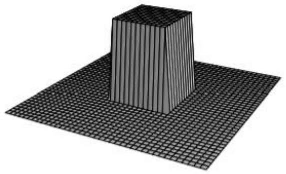
$I'[x, y]$

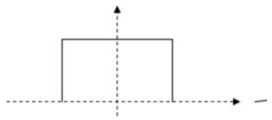
	0	10	20	30	30				

Box Filter

“box filter”

$^1/_9$	$^1/_9$	$^1/_9$
$^1/_9$	$^1/_9$	$^1/_9$
$^1/_9$	$^1/_9$	$^1/_9$





Input image

$I[x, y]$

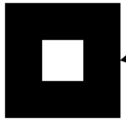
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Filtered image

$I'[x, y]$

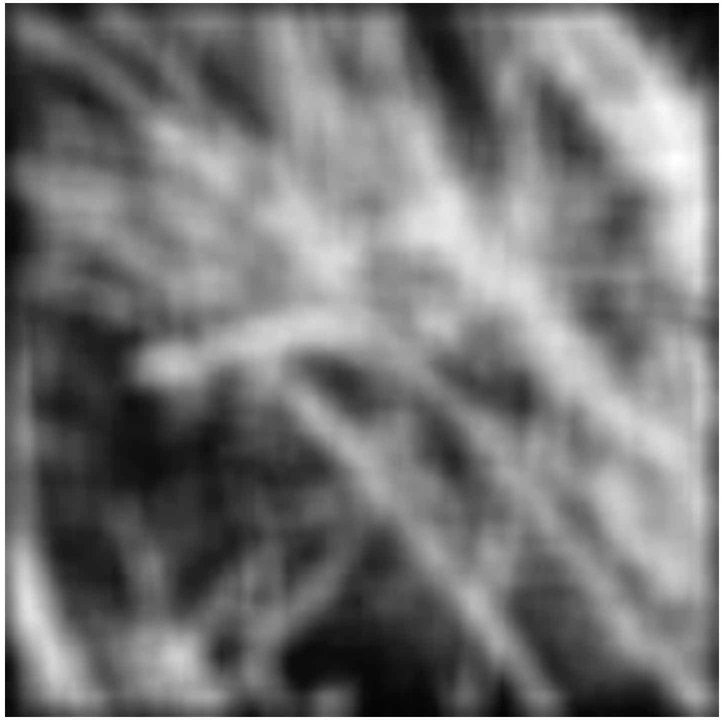
	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

Box Filter



Box filter:
white = max value, black = zero value

original



filtered

Gaussian Filter

What if we want **center pixels** to have **higher influence on the output**?

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$I[x,y]$

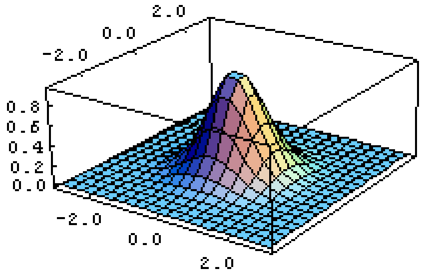
$\frac{1}{16}$

1	2	1
2	4	2
1	2	1

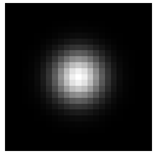
$H[u,v]$

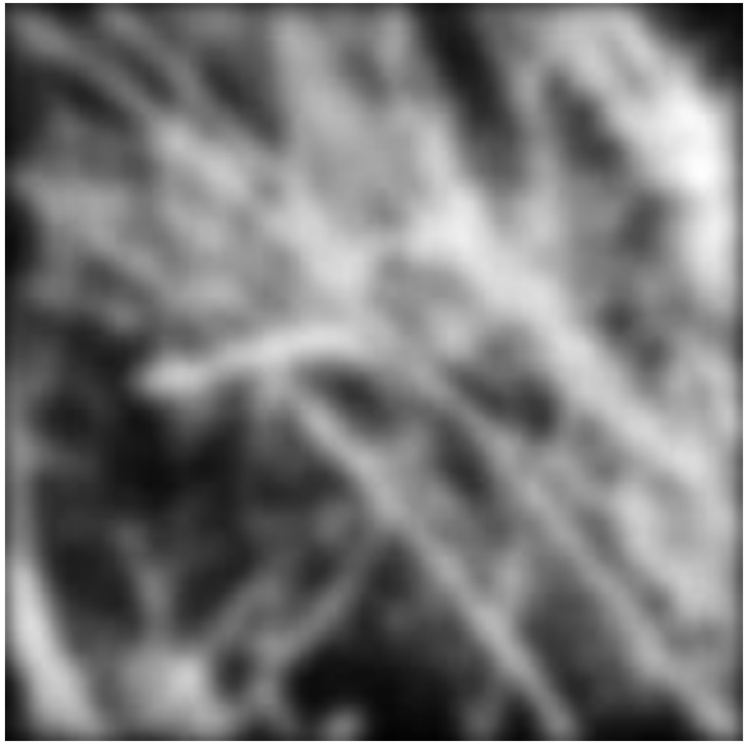
This kernel is the approximation of a Gaussian function:

$$H[u,v]\,\frac{1}{2\pi\sigma^2}\,e^{-\frac{u^2+v^2}{2\sigma^2}}$$

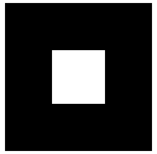


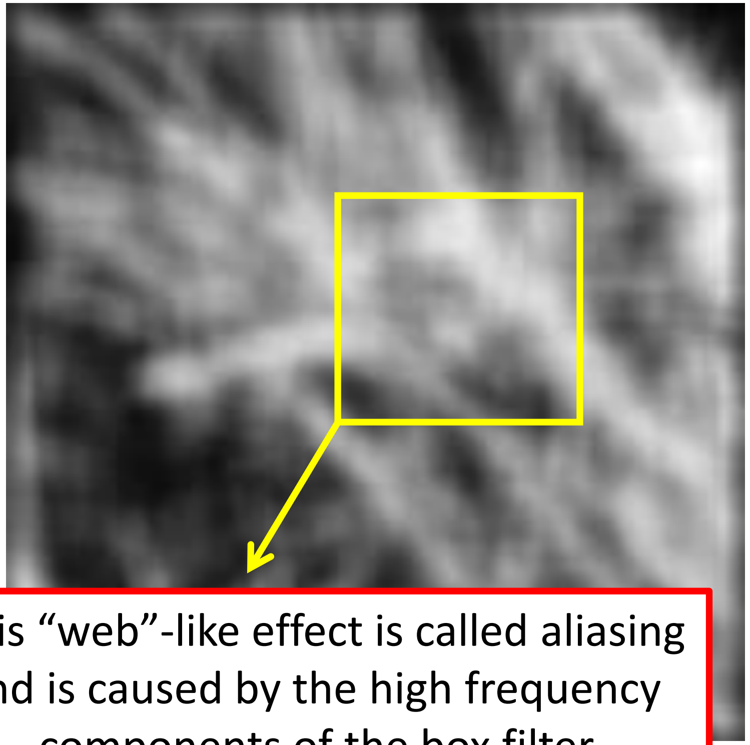
Gaussian Filter





Comparison with Box Filter





This “web”-like effect is called aliasing and is caused by the high frequency components of the box filter

Separable Filters

- **Box filter:**

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1\\ 1\\ 1\end{bmatrix} \cdot \frac{1}{3}\begin{bmatrix}1 & 1 & 1\end{bmatrix}$$

- **Gaussian filter:**

$$\frac{1}{16}\begin{bmatrix}1 & 2 & 1\\ 2 & 4 & 2\\ 1 & 2 & 1\end{bmatrix} = \frac{1}{4}\begin{bmatrix}1\\ 2\\ 1\end{bmatrix} \cdot \frac{1}{4}\begin{bmatrix}1 & 2 & 1\end{bmatrix}$$

- **Sobel filter:**

$$\begin{bmatrix}-1 & 0 & 1\\ -2 & 0 & 2\\ -1 & 0 & 1\end{bmatrix} = \begin{bmatrix}1\\ 2\\ 1\end{bmatrix} \cdot \begin{bmatrix}-1 & 0 & 1\end{bmatrix}$$

Separable Filters

- A convolution with a 2D filter of $w \times w$ pixel size requires $\boldsymbol{w^2}$ **multiply-add operations per pixel**
- 2D convolution can be sped up if the filter is **separable**, i.e., can be written as the product of two 1D filters (i.e., $H = v \cdot h^{\mathrm{T}}$): first perform a 1D horizontal convolution with h followed by a 1D vertical convolution with v:

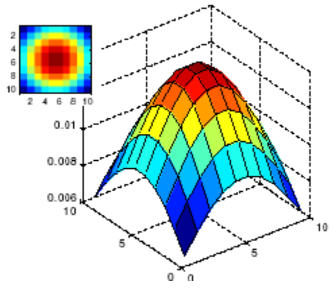
$$I' = I * H = (I * h^{\mathrm{T}}) * v$$

- Separable filters require only $\boldsymbol{2w}$ **multiply-add operations per pixel**
- **Box filters and Gaussian filters are separable**

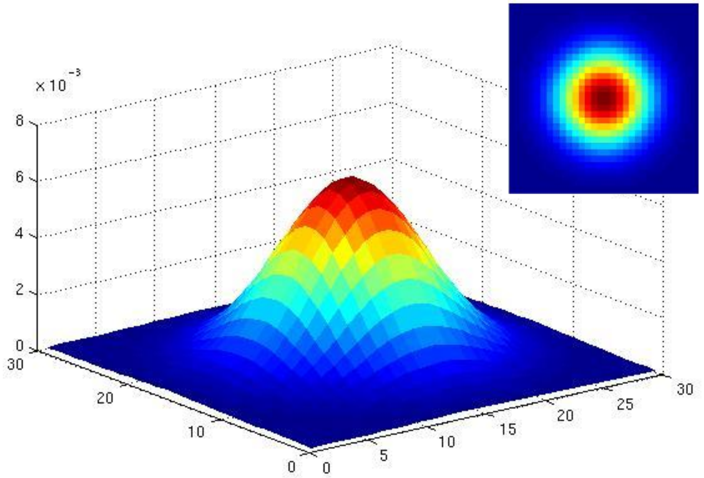
Gaussian Filter

What parameters matter?

- **Size** of the kernel
- NB: a Gaussian function has **infinite support**, but discrete filters use finite kernels



σ = 5 pixels
with 10×10 pixel kernel



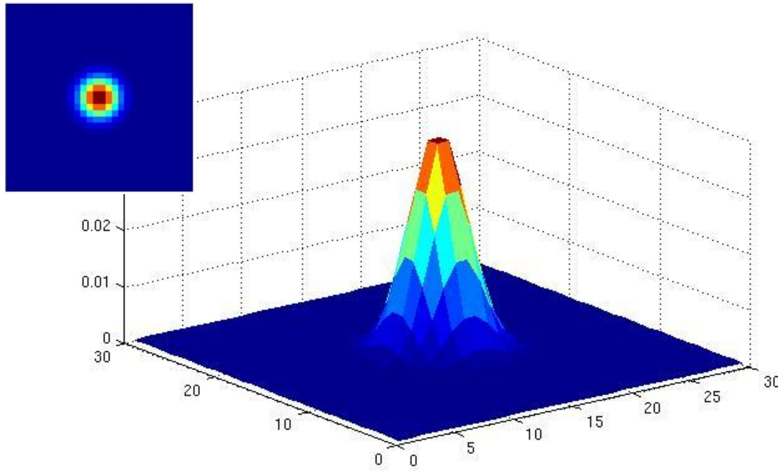
σ = 5 pixels
with 30×30 pixel kernel

Which one approximates better the ideal Gaussian filter, the left or the right one?

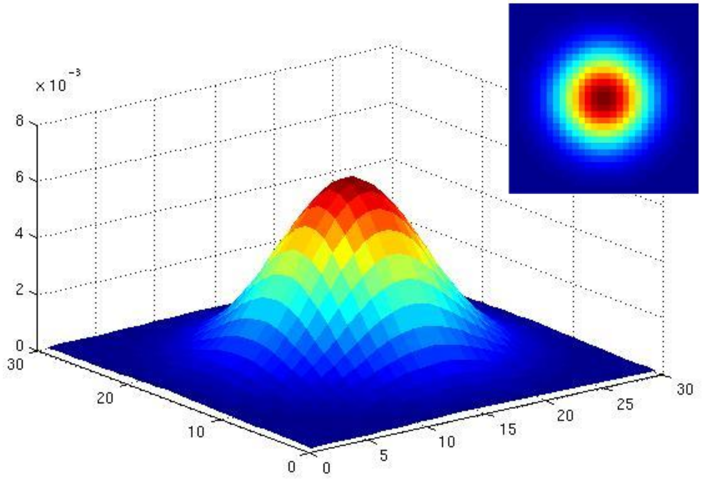
Gaussian Filter

What parameters matter?

- **Variance** of Gaussian: controls the amount of smoothing
- Recall: standard deviation = σ [pixels], variance = σ^2 [pixels^2]



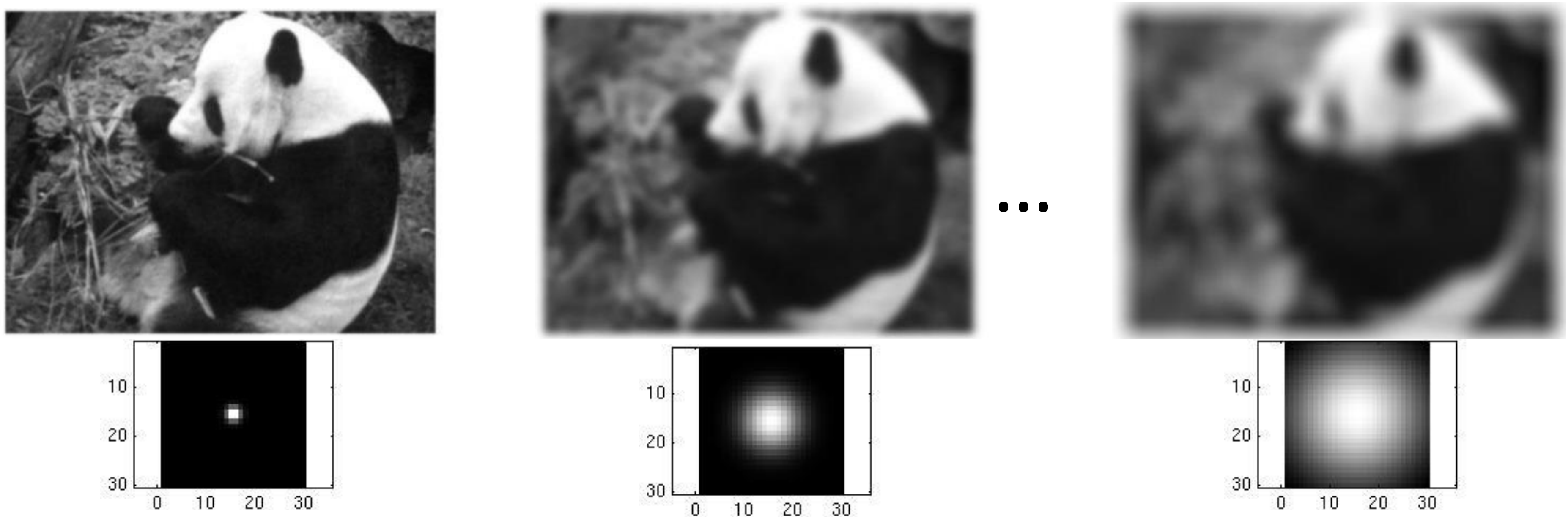
σ = 2 pixels
with 30×30 pixel kernel



σ = 5 pixels
with 30×30 pixel kernel

Gaussian Filter

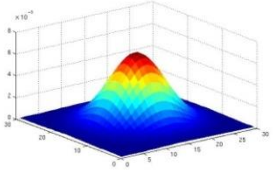
σ is called "**scale**" of the Gaussian kernel, and **controls the amount of smoothing**.



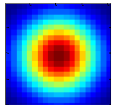
Sample Matlab code

```
>> hsize = 20;
>> sigma = 5;
>> h = fspecial('gaussian', hsize, sigma);
```

```
>> mesh(h);
```



```
>> imagesc(h);
```



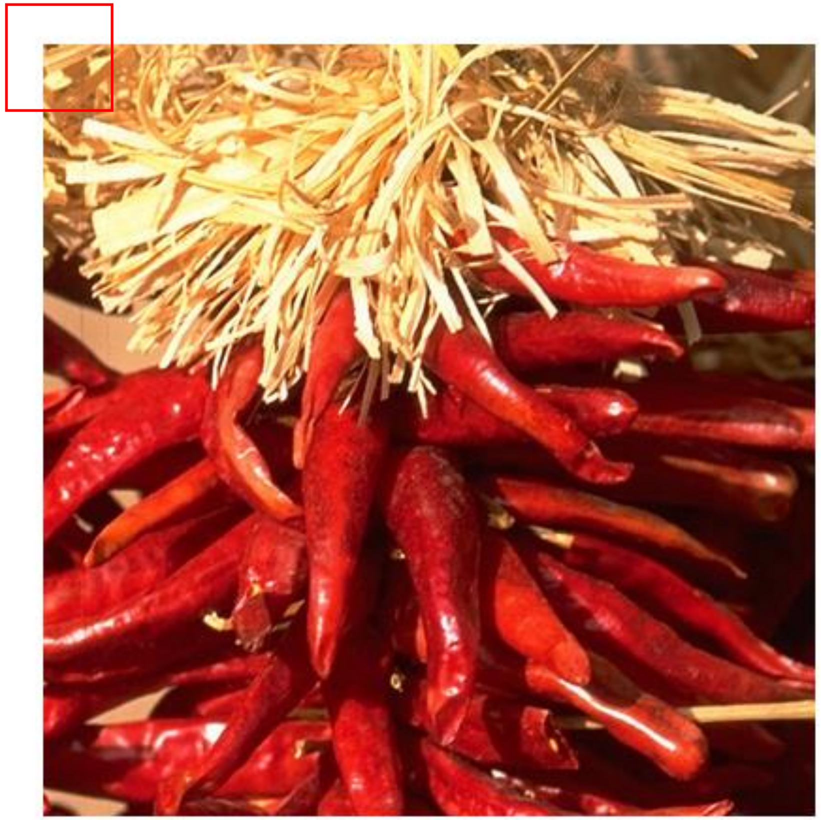
```
>> im = imread('panda.jpg');
>> outim = imfilter(im, h);
>> imshow(outim);
```

im

outim

Boundary issues

- What about near the image edges?
  - the filter window falls off the edges of the image
  - need to pad the image borders
  - methods:



Boundary issues

- What about near the image edges?
  - the filter window falls off the edges of the image
  - need to pad the image borders
  - methods:
    - zero padding (black)

Boundary issues

- What about near the image edges?
  - the filter window falls off the edges of the image
  - need to pad the image borders
  - methods:
    - zero padding (black)
    - wrap around

Boundary issues

- What about near the image edges?
  - the filter window falls off the edges of the image
  - need to pad the image borders
  - methods:
    - zero padding (black)
    - wrap around
    - copy edge

Boundary issues

- What about near the image edges?
  - the filter window falls off the edges of the image
  - need to pad the image borders
  - methods:
    - zero padding (black)
    - wrap around
    - copy edge
    - reflect across edge

Summary on (linear) smoothing filters

- Smoothing filter
  - has **positive values** (also called coefficients)
  - **sums to 1** → preserve brightness of constant regions
  - **removes "high-frequency"** components; "low-pass" filter

Today’s Outline

- Low-pass filtering
  - Linear filters
  - Non-linear filters
- Edge Detection
  - Canny edge detector

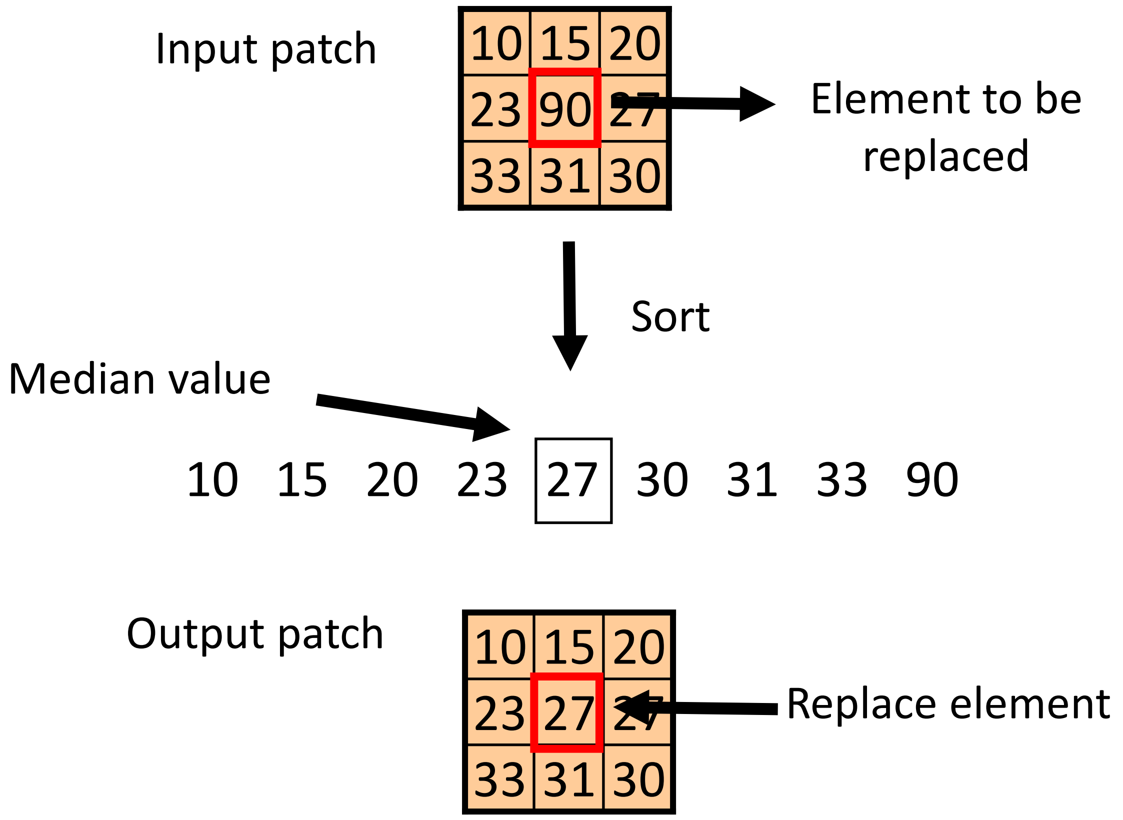
Effect of smoothing filters

Linear smoothing filters do not alleviate salt and pepper noise!

Median Filter

- It is a **non-linear filter**

- **Removes spikes**:
  good for "*impulse noise*"
  and "*salt & pepper noise*"



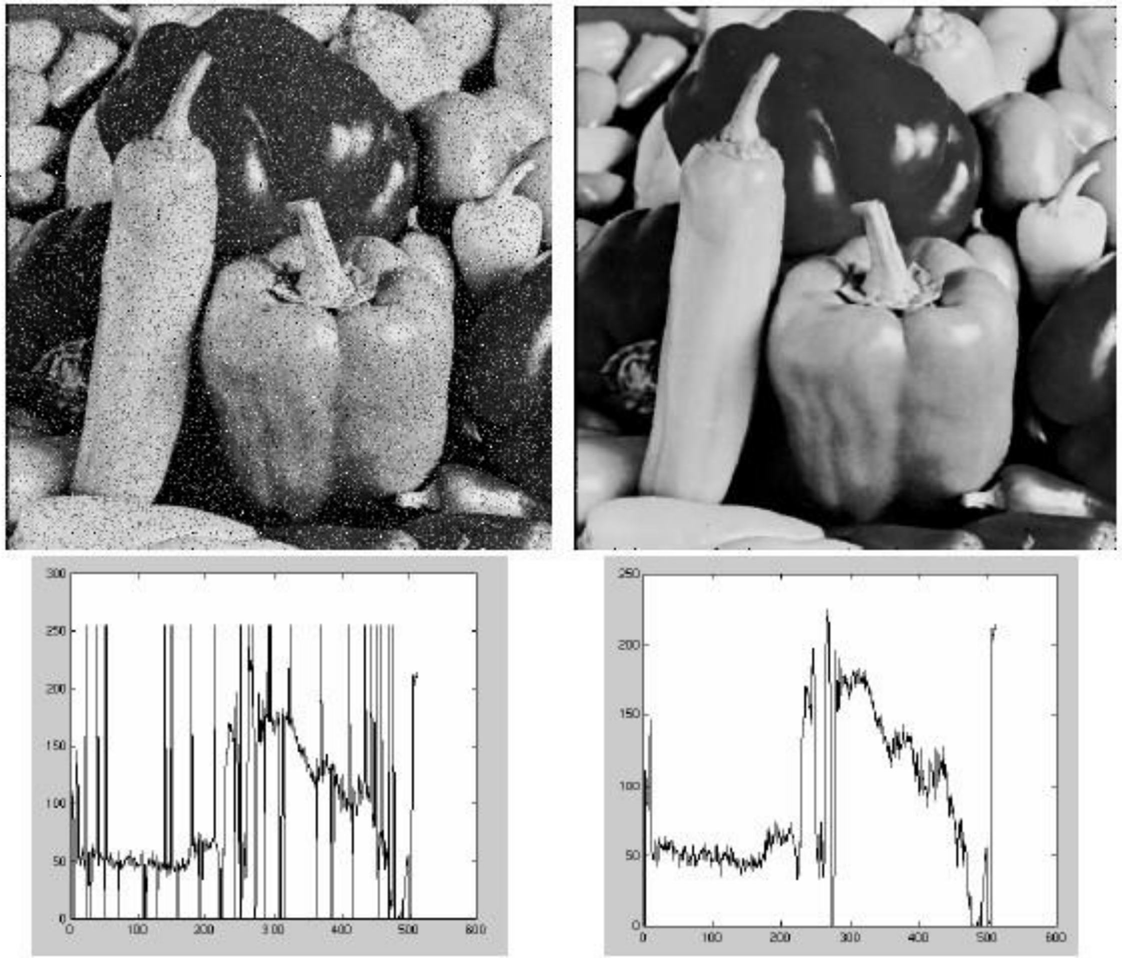
Median Filter

- It is a **non-linear filter**

- **Removes spikes**:
  good for "*impulse noise*"
  and "*salt & pepper noise*"

Salt and
pepper noise

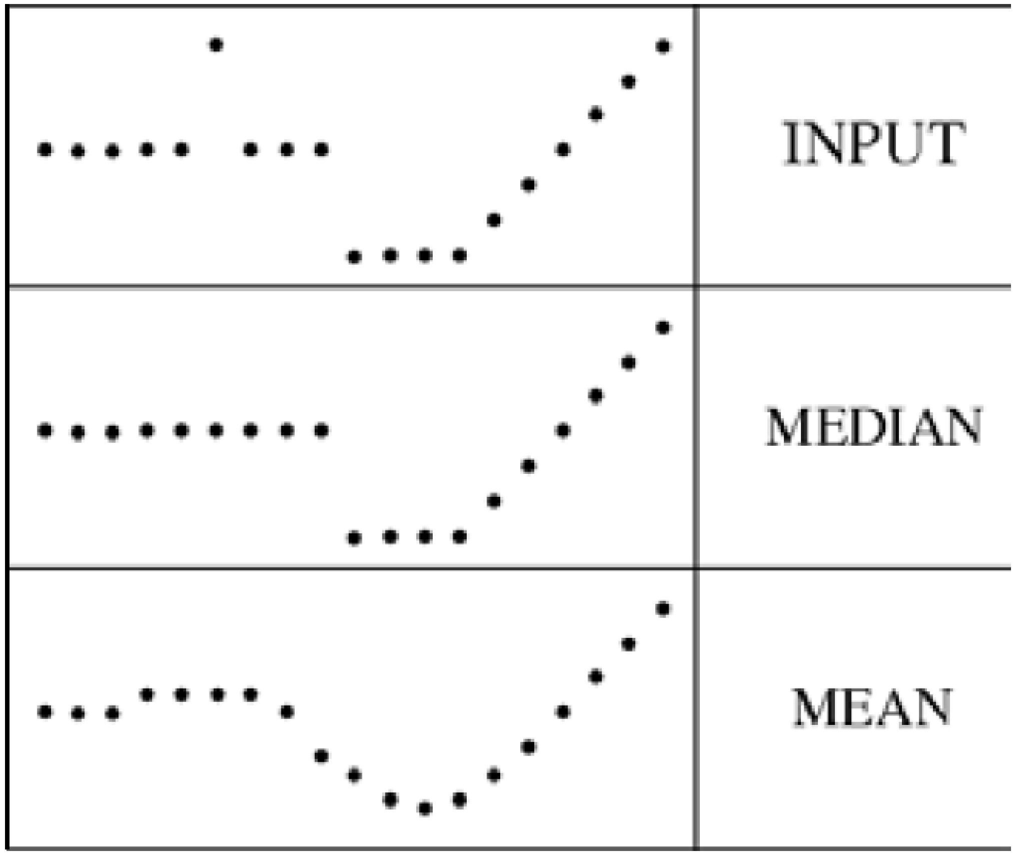
Median
filtered



Plots of one row of the image

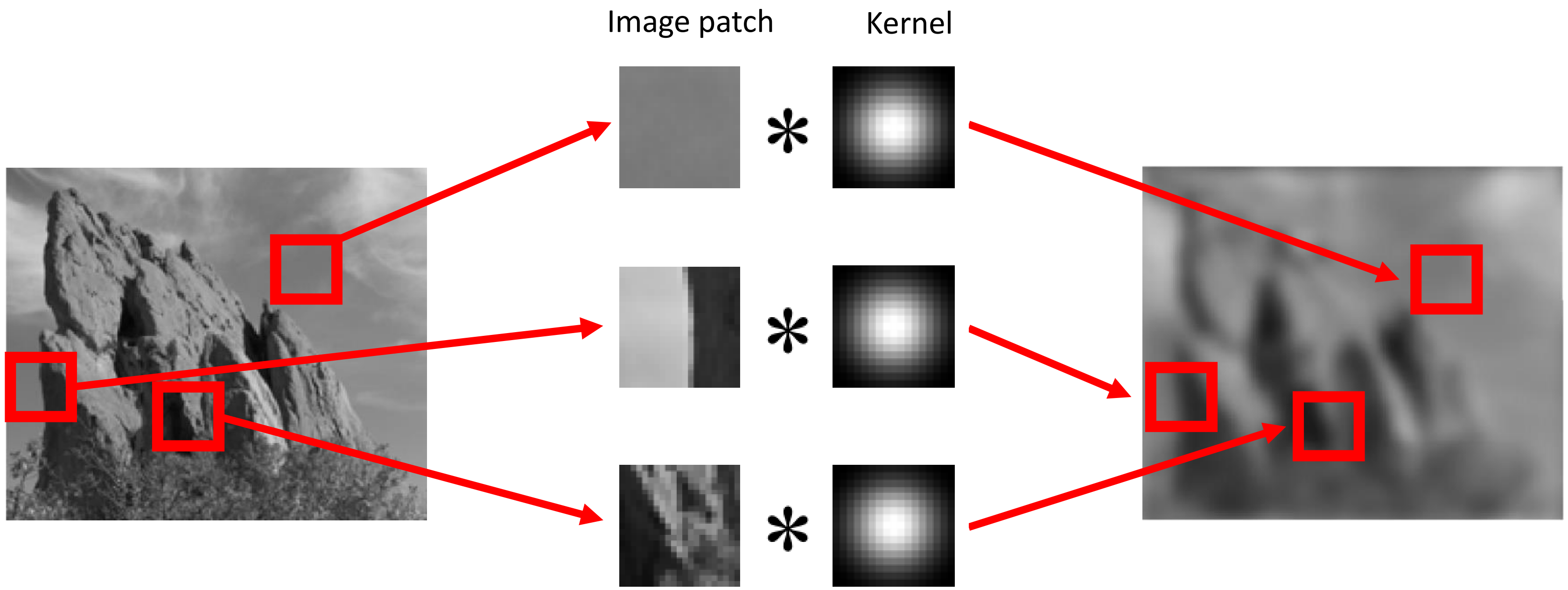
Median Filter

- It is a **non-linear filter**
- **Removes spikes**:
  good for "*impulse noise*"
  and "*salt & pepper noise*"
- Differently from linear filters,
  it **preserves strong edges**



Gaussian vs. Median Filter

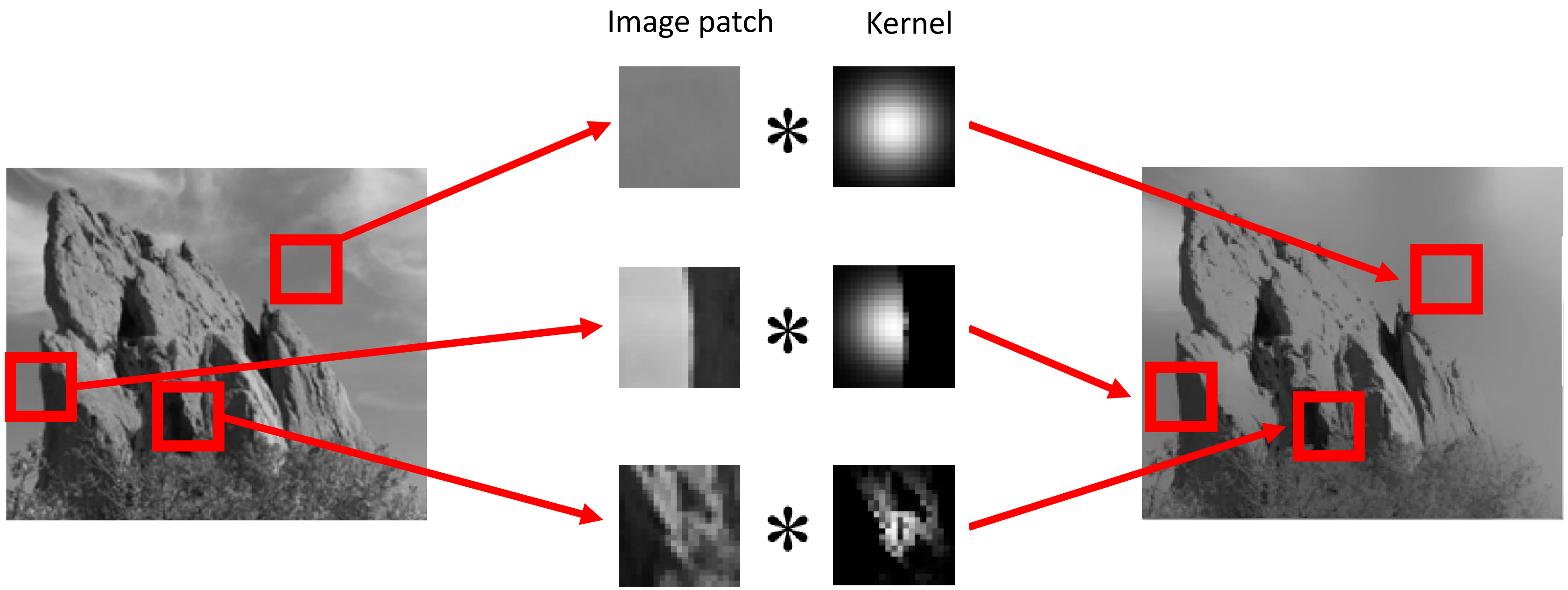
- **Gaussian filters do not preserve strong egdes (discontinuites)**. This is because they apply the same kernel everywhere.
- **Median filters** do **preserve strong edges but don't smooth** as good as Gaussian filters with **Gaussian noise**.



Gaussian filter

Bilateral Filter

- **Bilateral filters** solve this by adapting the kernel locally to the intensity profile, so they are **patch-content dependent**
- Bilateral filters only **smooth pixels with brightness similar to the center pixel** and ignore **influence of pixels with different brightness across the discontinuity**

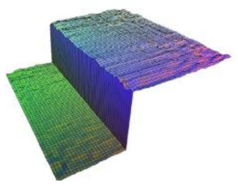


Bilateral filter

Bilateral Filter

1D Gaussian range component | spatial component

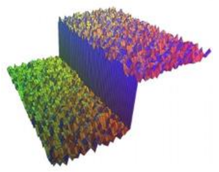
$$I'[x,y] = \frac{1}{W_p[x,y]} \sum_{u=-k}^{k} \sum_{v=-k}^{k} I[x-u,y-v]\, G_{\sigma_r}(I[x-u,y-v]-I[x,y])\, G_{\sigma_s}[u,v]$$



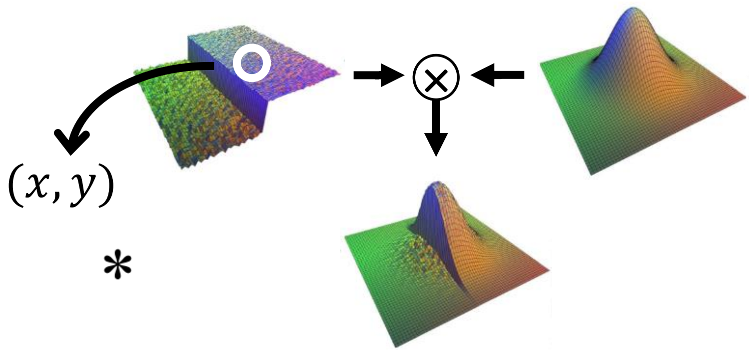
smoothed image

$$= \frac{1}{W_p[x,y]} \cdot$$

normalization factor



input image



adaptive filter

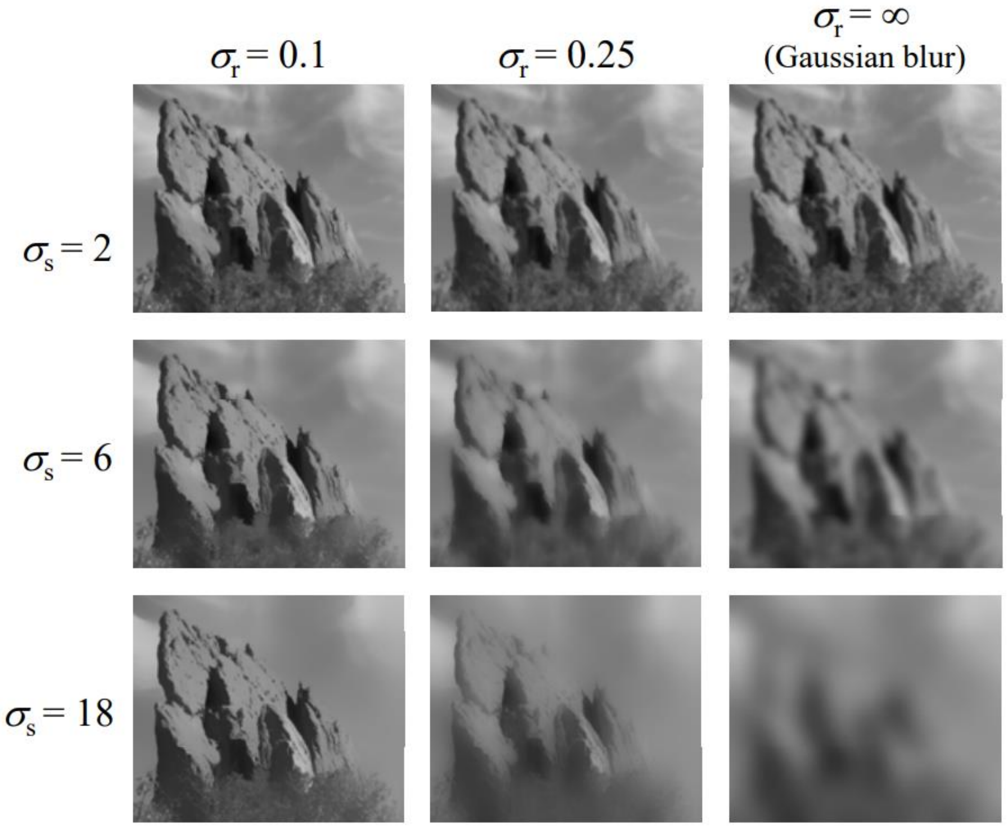
$$W_p[x,y] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} G_{\sigma_r}(I[x-u,y-v]-I[x,y])\, G_{\sigma_s}[u,v]$$

Normalization factor
(so that the filter values sum to 1)

Bilateral Filter

Stronger edges are
are smoothed

larger neighborhoods
are smoothed

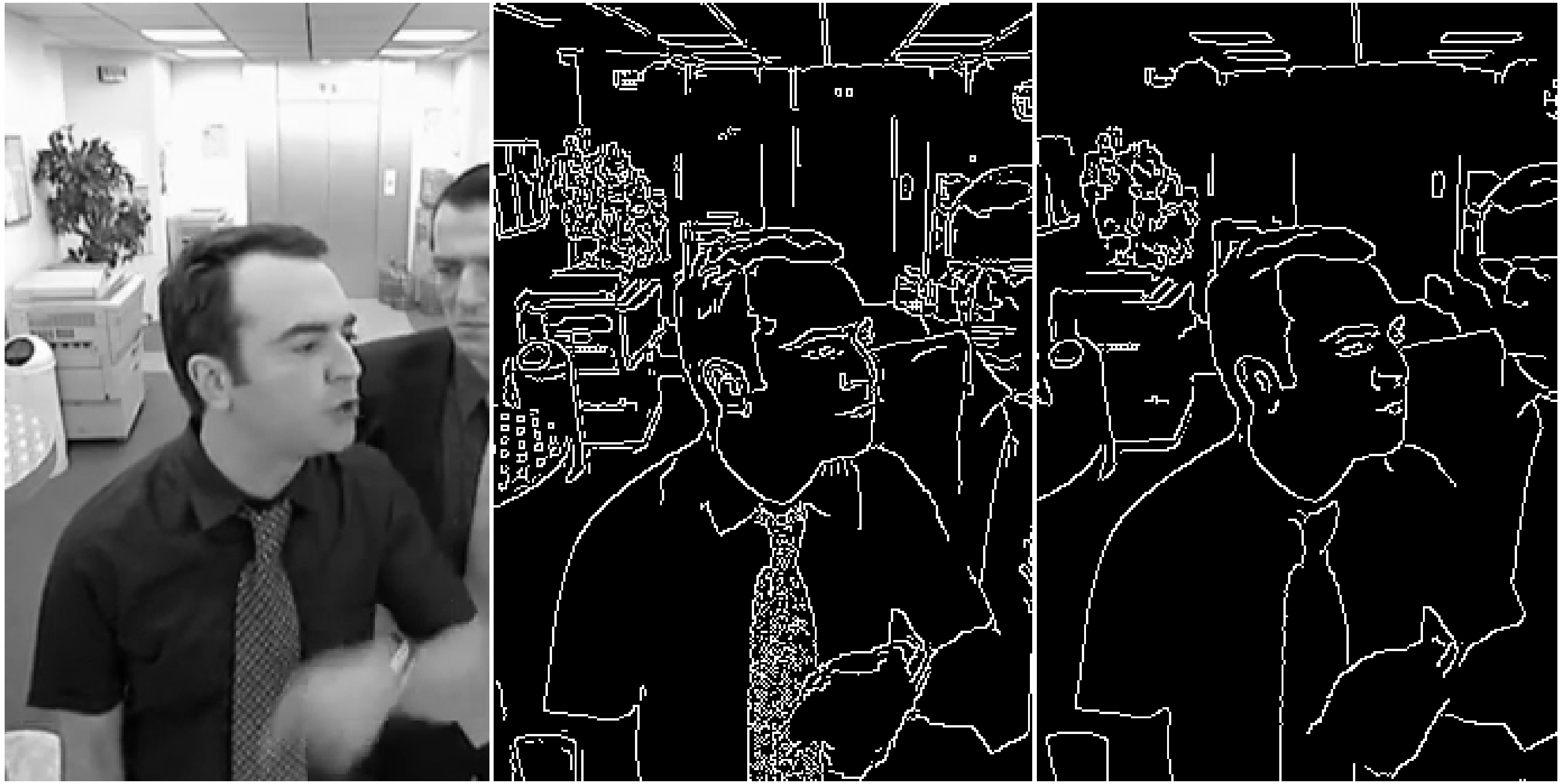


Today’s Outline

- Low-pass filtering
  - Linear filters
  - Non-linear filters
- Edge Detection
  - Canny edge detector

Edge Detection

- Goal: to find the boundaries (edges) of objects within images



Edge Detection

- Edges look like steep cliffs in the $I(x,y)$ function

Original image $I(x, y)$

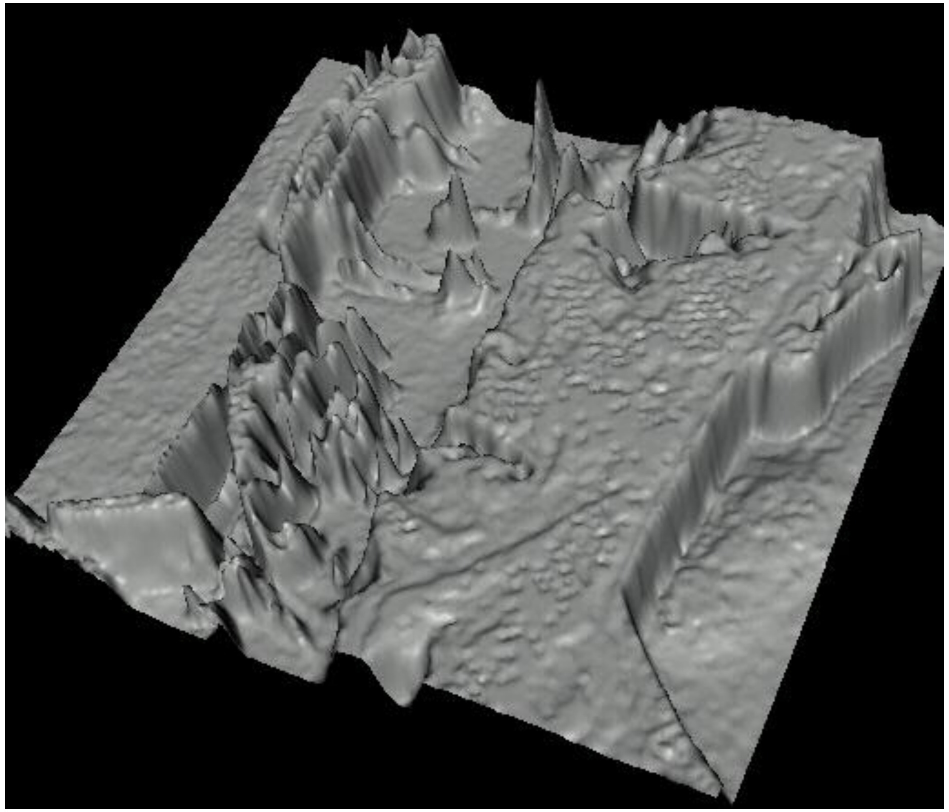
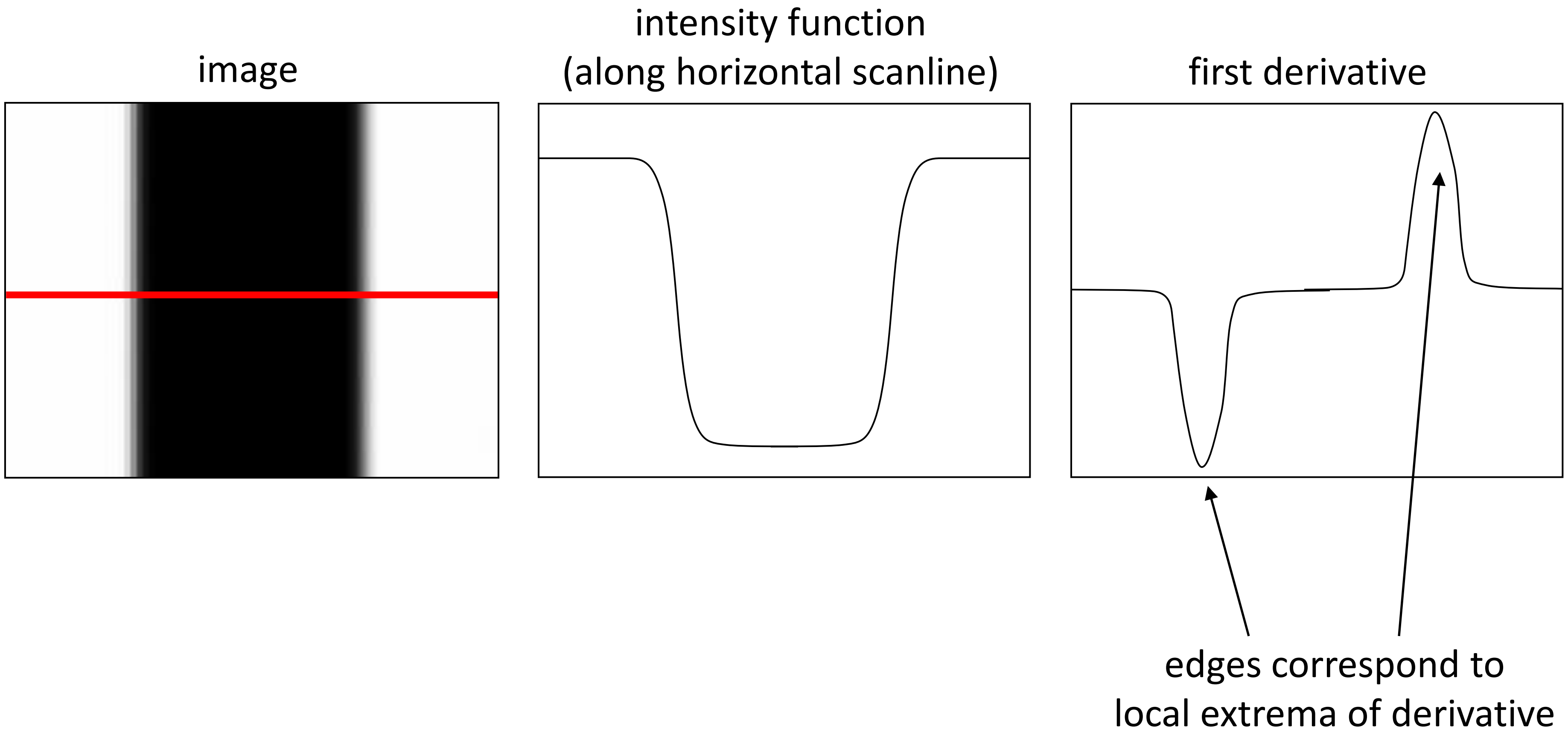


Image plotted as $I(x, y)$ function

Derivatives and Edges

- An edge is a place of fast change in the image intensity function



Differentiation and Convolution

- For a continuous function $I(x, y)$ the partial derivative along x is:

$$\frac{\partial I(x,y)}{\partial x} = \lim_{\varepsilon \to 0} \frac{I(x+\varepsilon, y) - I(x,y)}{\varepsilon}$$

- For a discrete function, we can use **adjacent or central** finite differences:

$$\frac{\partial I(x,y)}{\partial x} \approx \frac{I(x+1, y) - I(x,y)}{1} \quad \text{or} \quad \frac{\partial I(x,y)}{\partial x} \approx \frac{I(x+1, y) - I(x-1,y)}{2}$$

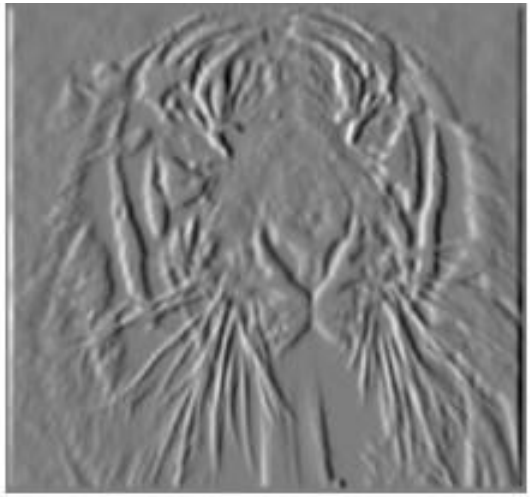
What would be the respective filters along x and y to implement the partial derivatives as a convolution?

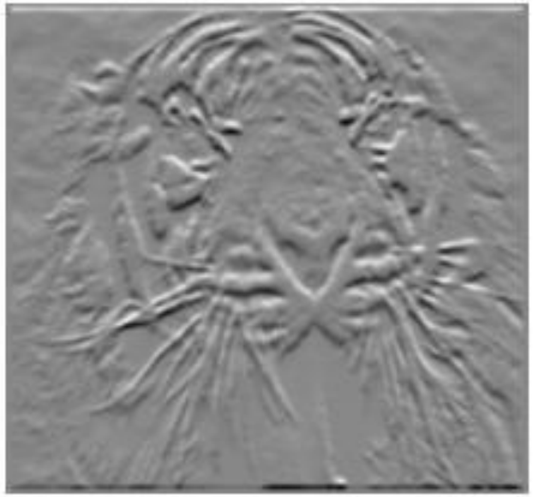
Partial Derivatives using Adjacent Differences

$$\frac{\partial I(x,y)}{\partial x}$$

$$\frac{\partial I(x,y)}{\partial y}$$

-1	1





-1
1

Partial Derivatives using Central Differences

Prewitt filter $G_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$, $G_y = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

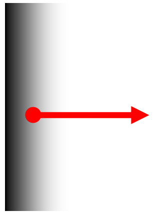
Sobel filter $G_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$, $G_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$

```
Sample Matlab code
>> im = imread('lion.jpg');
>> h = fspecial('sobel');
>> outim = imfilter(double(im), h);
>> imagesc(outim);
>> colormap gray;
```

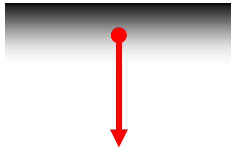
Image Gradient

- The **image gradient**: $\nabla \mathrm{I} = \left[\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\right]$

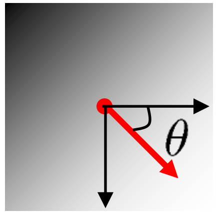
- The gradient points in the **direction of steepest ascent**:



$\nabla \mathrm{I} = \left[\frac{\partial I}{\partial x}, 0\right]$



$\nabla \mathrm{I} = \left[0, \frac{\partial I}{\partial y}\right]$



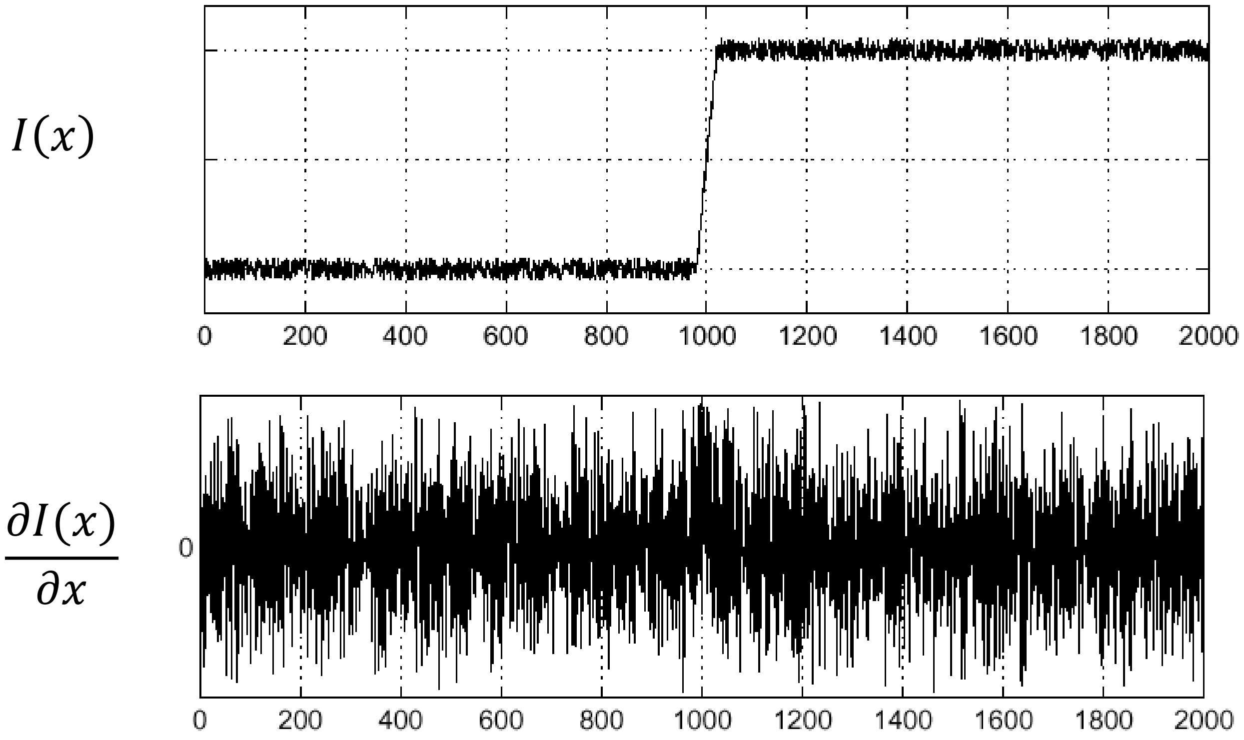
$\nabla \mathrm{I} = \left[\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\right]$

- The **gradient direction** (perpendicular to the edge) is given by: $\theta = atan2\left(\frac{\partial I}{\partial y}, \frac{\partial I}{\partial x}\right)$

- The **edge strength** is given by the gradient magnitude $\|\nabla I\| = \sqrt{\left(\frac{\partial I}{\partial x}\right)^2 + \left(\frac{\partial I}{\partial y}\right)^2}$

Effects of Noise

- Consider a single row or column of the image



Where is the edge?

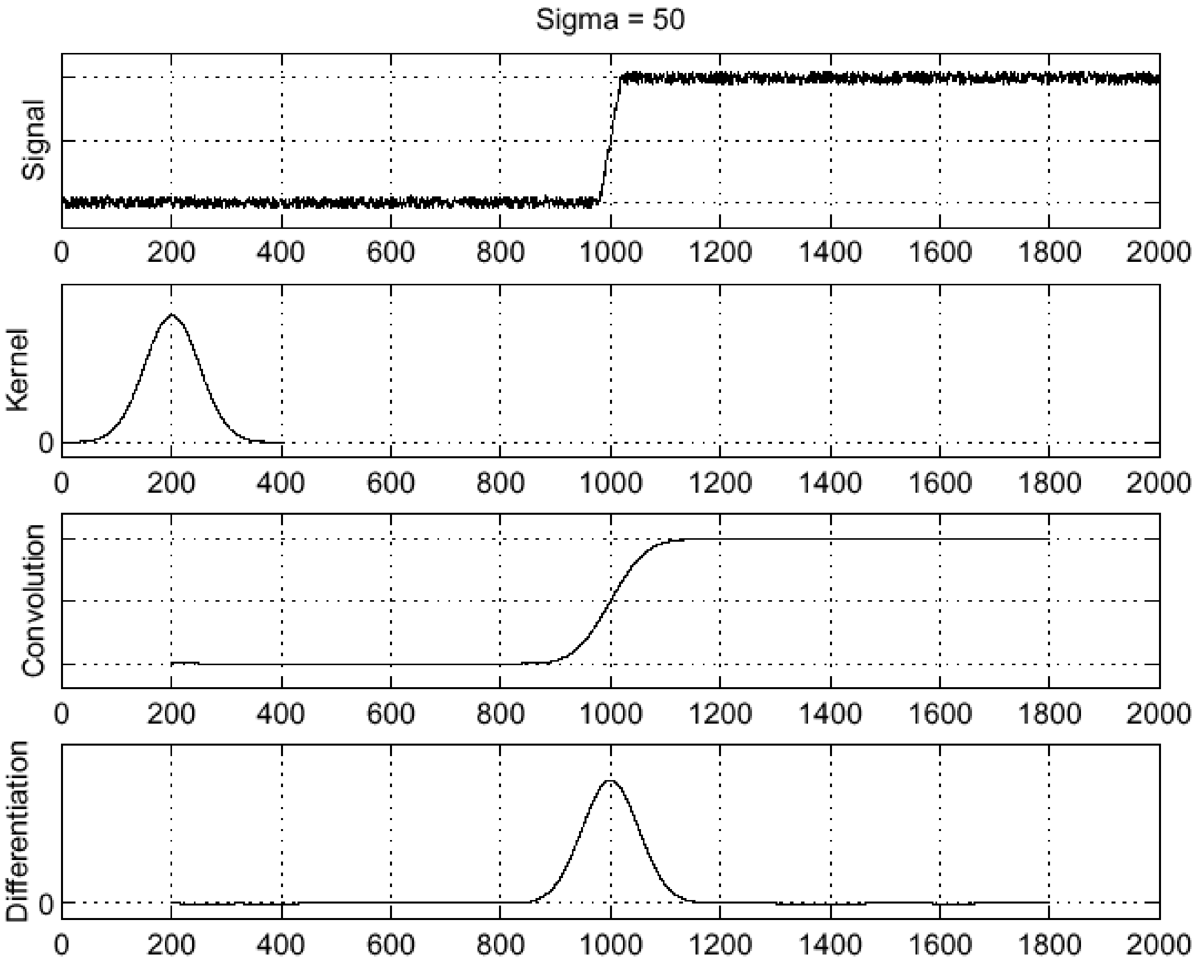
Solution: smooth first

I

H

$I * H$

$$\frac{\partial}{\partial x}(I * H)$$



Location of edges: **look for peaks** in $\frac{\partial}{\partial x}(I * H)$

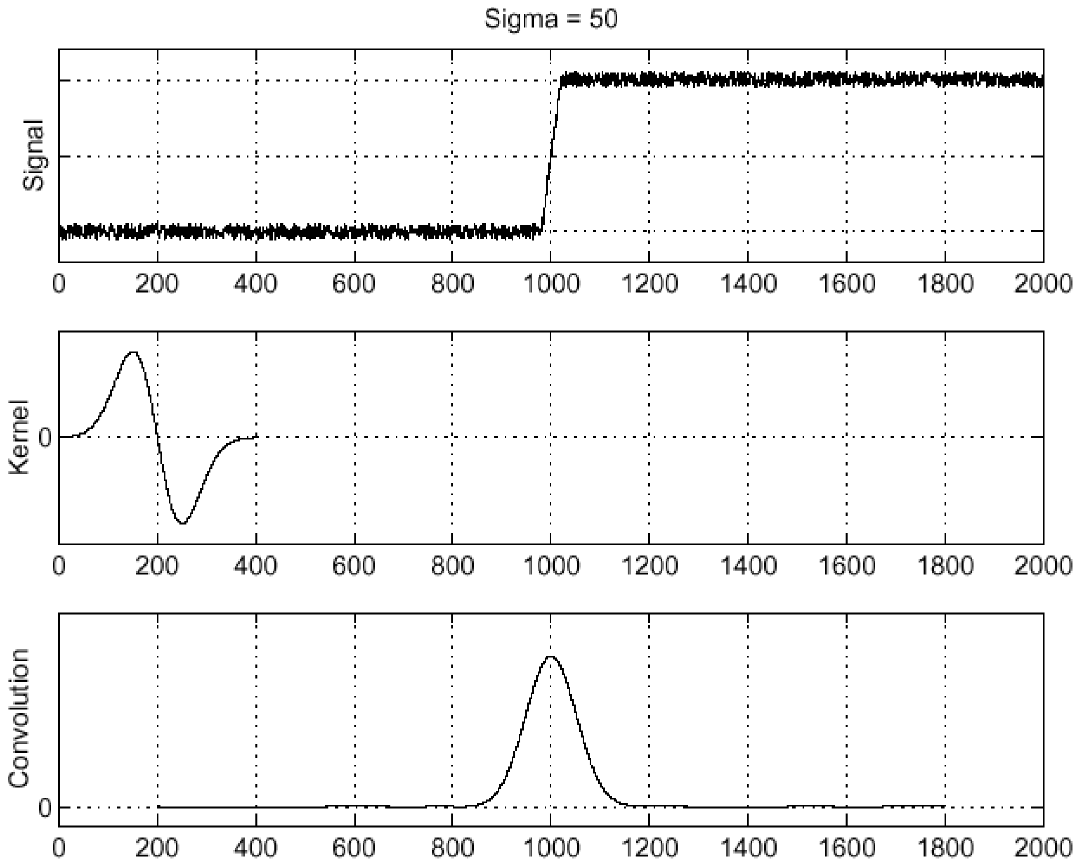
Alternative: combine derivative and smoothing filter

- Differentiation property of convolution: $$\frac{\partial}{\partial x}(I * H) = I * \frac{\partial H}{\partial x}$$

I

$\frac{\partial H}{\partial x}$

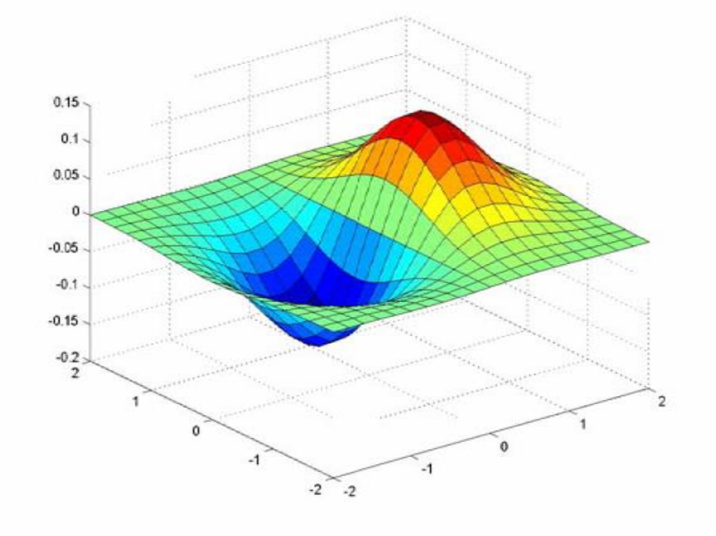
$I * \frac{\partial H}{\partial x}$



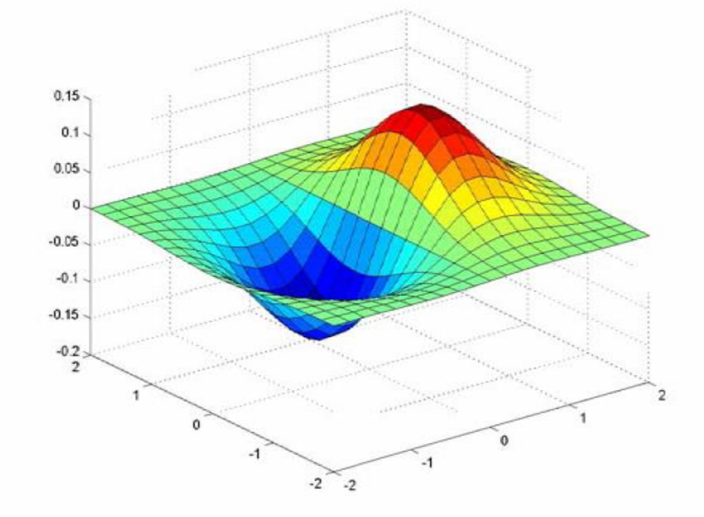
Derivative of Gaussian filter G along x

$$(I * G) * H = I * (G * H)$$

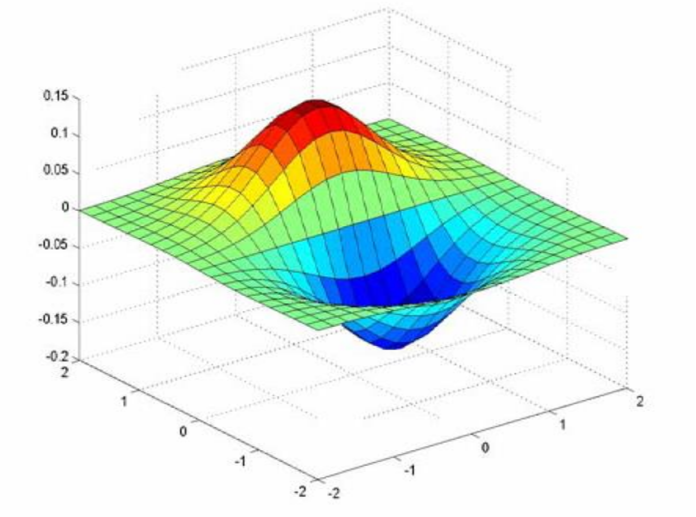
$$\begin{bmatrix} 0.0030 & 0.0133 & 0.0219 & 0.0133 & 0.0030 \\ 0.0133 & 0.0596 & 0.0983 & 0.0596 & 0.0133 \\ 0.0219 & 0.0983 & 0.1621 & 0.0983 & 0.0219 \\ 0.0133 & 0.0596 & 0.0983 & 0.0596 & 0.0133 \\ 0.0030 & 0.0133 & 0.0219 & 0.0133 & 0.0030 \end{bmatrix} * \begin{bmatrix} -1 & 1 \end{bmatrix}$$



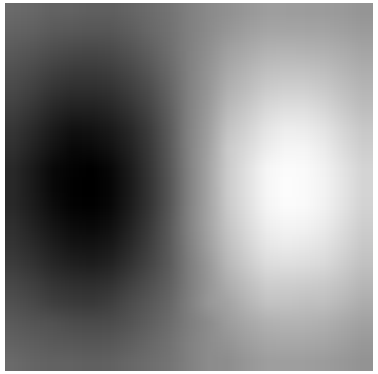
Derivative of Gaussian Filters

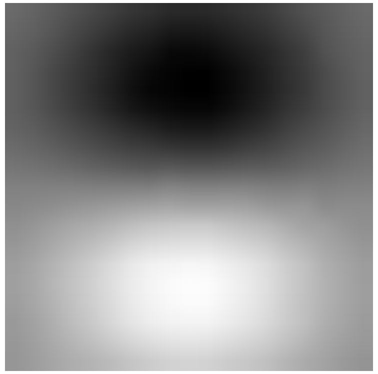


x-direction



y-direction





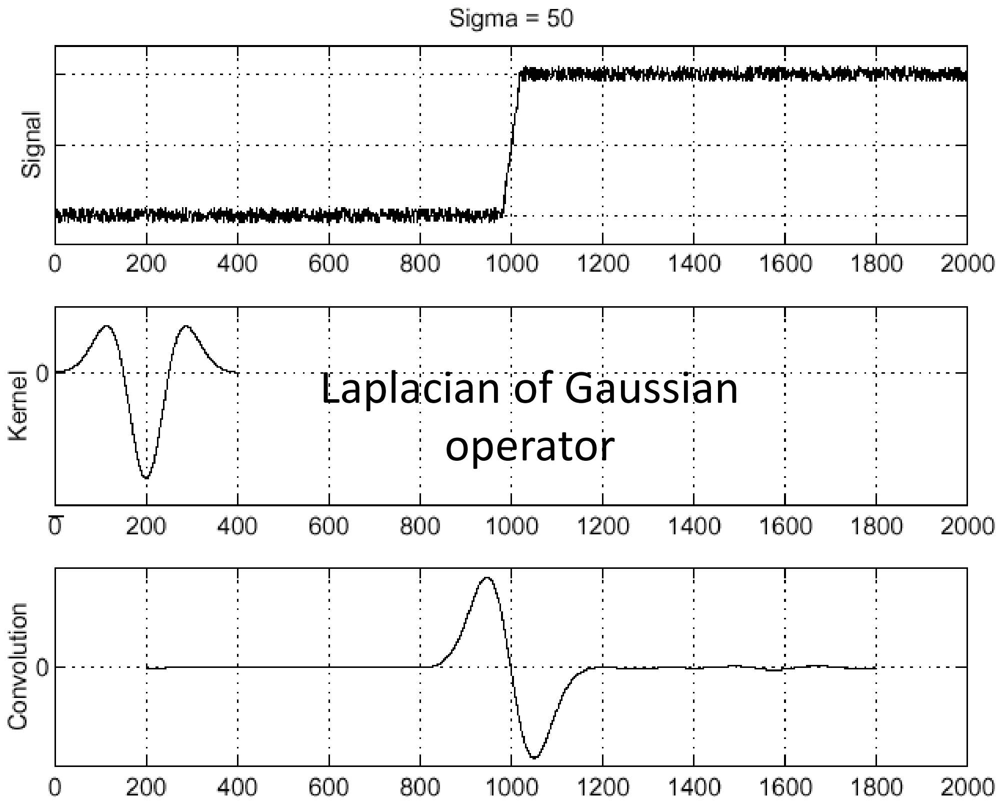
Laplacian of Gaussian

$$\frac{\partial^2}{\partial x^2}(I * H) = I * \frac{\partial^2 H}{\partial x^2}$$

$I(x)$

$\frac{\partial^2 H}{\partial x^2}$

$I * \frac{\partial^2 H}{\partial x^2}$

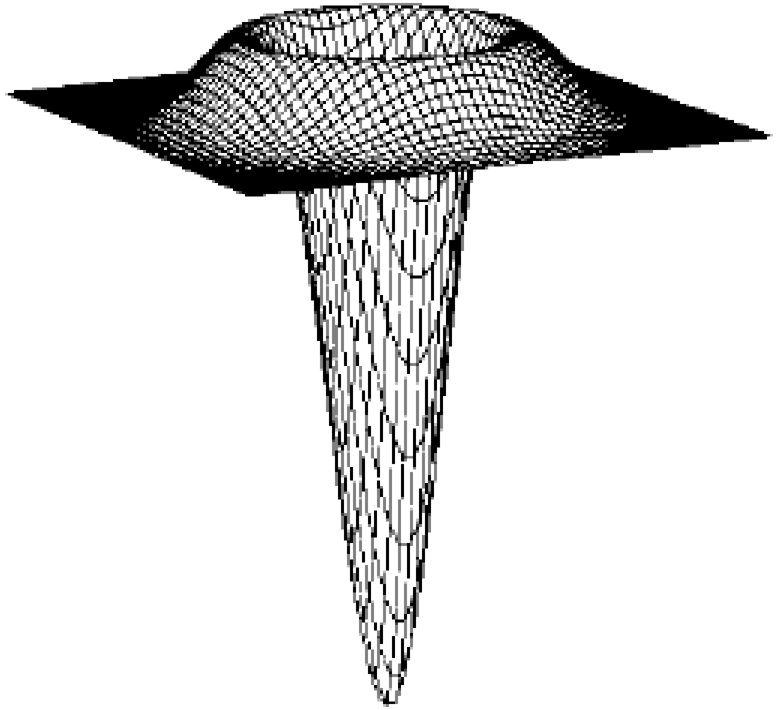


Location of edges: look for **Zero-crossings** of $I * \frac{\partial^2 H}{\partial x^2}$

Laplacian of Gaussian (LoG)

- The Laplacian of Gaussian is a circularly symmetric filter defined as:

$$\nabla^2 G_\sigma = \frac{\partial^2 G_\sigma}{\partial x^2} + \frac{\partial^2 G_\sigma}{\partial y^2}$$



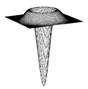
∇^2 is the Laplacian operator: $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

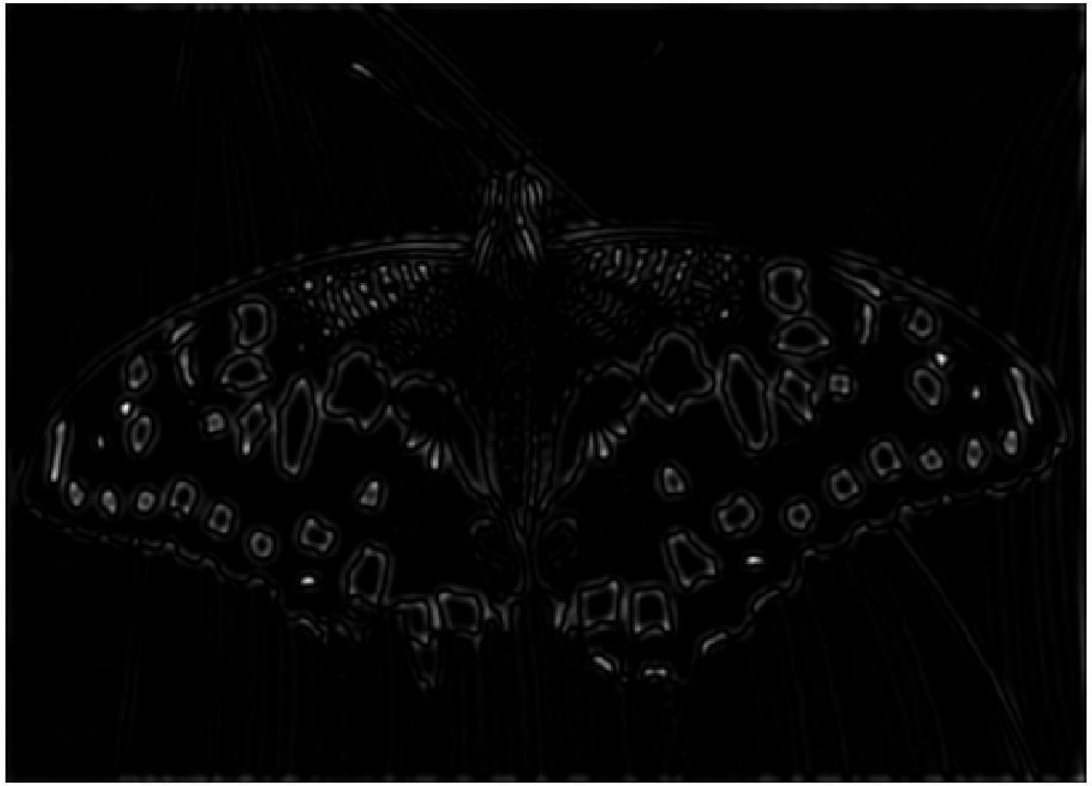
- Two commonly used approximations of LoG filter:

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Example: Convolving an Image with $\nabla^2 G_\sigma$

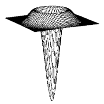
Example: Convolving an Image with $\nabla^2 G_\sigma$

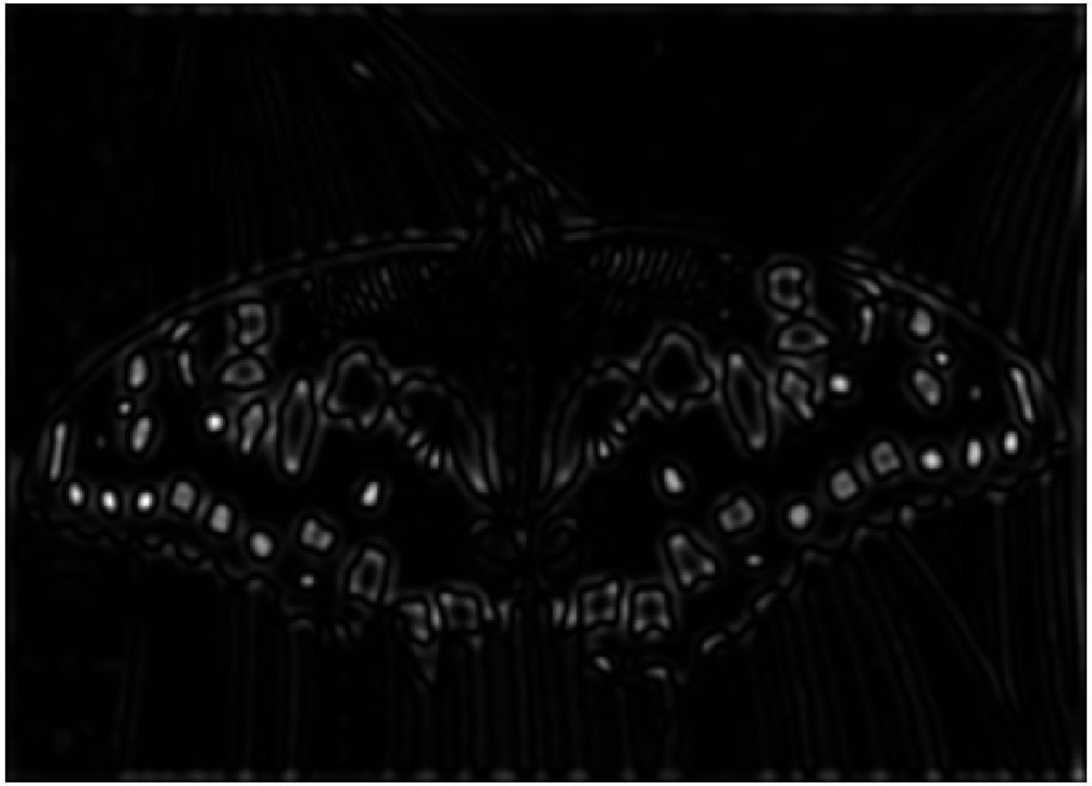
- 



sigma = 2

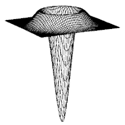
Example: Convolving an Image with $\nabla^2 G_\sigma$

- 



sigma = 3.1296

Example: Convolving an Image with $\nabla^2 G_\sigma$

- 

sigma = 4.8972

Summary on Linear Filters

- Smoothing filter
  - has **positive values** (also called coefficients)
  - **sums to 1** → preserve brightness of constant regions
  - **removes "high-frequency"** components; "low-pass" filter

- Derivative filter:
  - **has opposite signs** used to get high response in regions of high contrast
  - **sums to 0** → no response in constant regions
  - **highlights "high-frequency"** components: "high-pass" filter

Today's Outline

- Low-pass filtering
  - Linear filters
  - Non-linear filters
- Edge Detection
  - Canny edge detector

The Canny Edge-Detection Algorithm (1986)

Despite invented in 1986, the Canny edge detector is still the most popular edge detection algorithm today

This image is called **Lenna image** and was a standard benchmark in edge detection and image processing: https://en.wikipedia.org/wiki/Lenna

Canny, J., A Computational Approach To Edge Detection, IEEE Transactions on Pattern Analysis and Machine Intelligence, (T-PAMI), 1986. PDF.

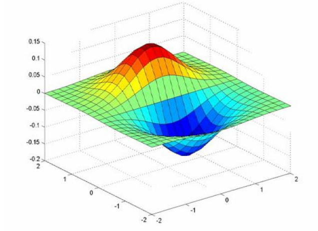
The Canny Edge-Detection Algorithm (1986)

1. **Take a grayscale image**. If RGB, convert it into a grayscale $I(x, y)$ by replacing each pixel by the average value of its R, G, B components.

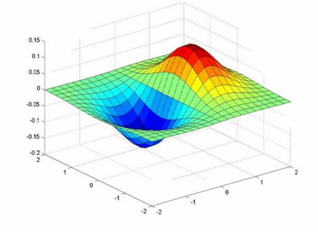
Canny, J., A Computational Approach To Edge Detection, IEEE Transactions on Pattern Analysis and Machine Intelligence, (T-PAMI), 1986. PDF.

The Canny Edge-Detection Algorithm (1986)

2. **Convolve the image** I with x and y derivatives of Gaussian filter and compute the edge strength $||\nabla I||$

$\frac{\partial G_\sigma}{\partial x}$ 

$$\frac{\partial I}{\partial x} = I * \frac{\partial G_\sigma}{\partial x}$$

$\frac{\partial G_\sigma}{\partial y}$ 

$$\frac{\partial I}{\partial y} = I * \frac{\partial G_\sigma}{\partial y}$$

Edge strength: $||\nabla I|| = \sqrt{\left(\frac{\partial I}{\partial x}\right)^2 + \left(\frac{\partial I}{\partial y}\right)^2} \rightarrow$

Canny, J., A Computational Approach To Edge Detection, IEEE Transactions on Pattern Analysis and Machine Intelligence, (T-PAMI), 1986. PDF.

The Canny Edge-Detection Algorithm (1986)

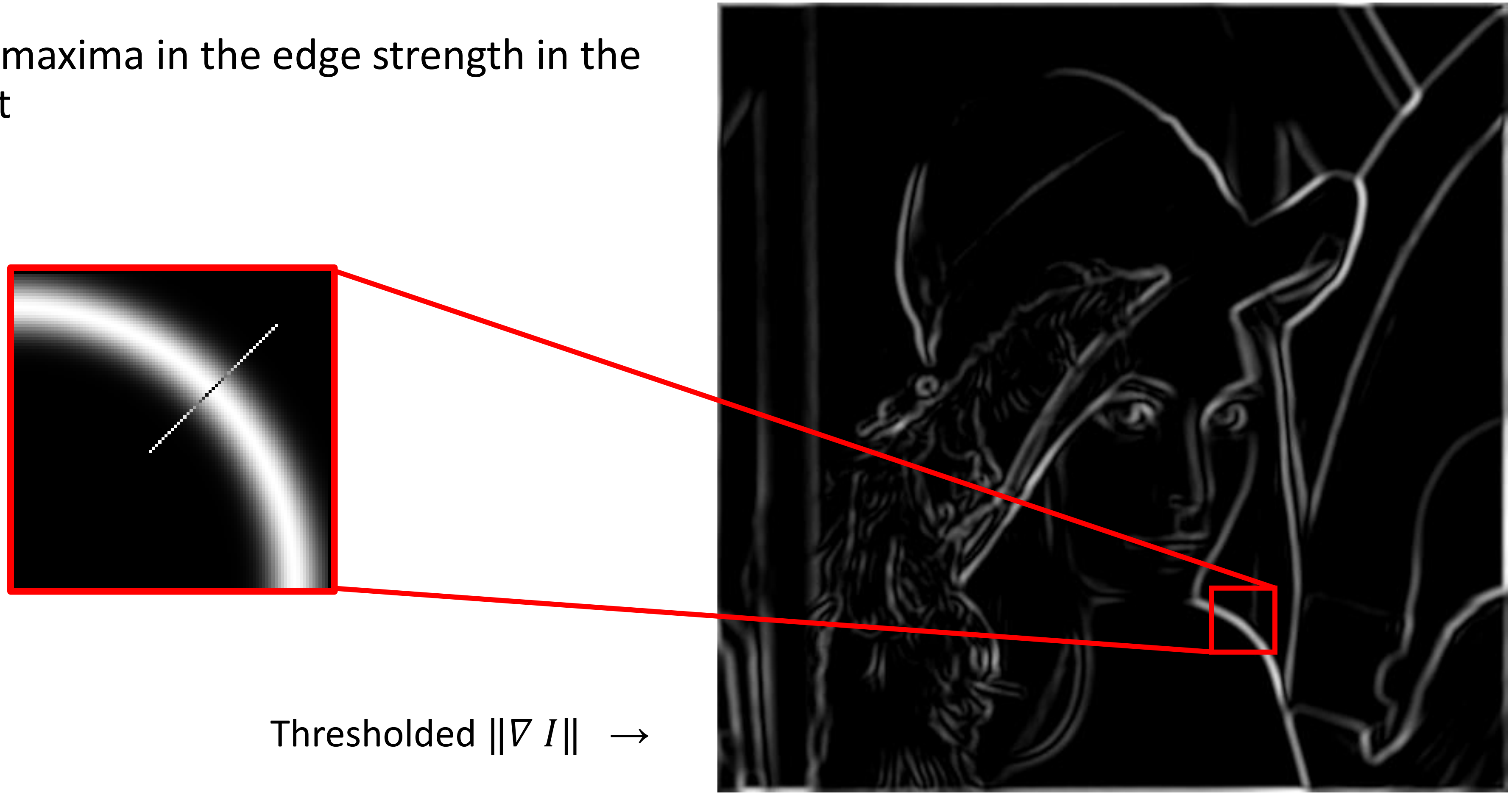
3. **Thresholding**: set to 0 all pixels of $||\nabla I||$ whose value is below a given threshold

Thresholded $||\nabla I||$ $\rightarrow$

Canny, J., A Computational Approach To Edge Detection, IEEE Transactions on Pattern Analysis and Machine Intelligence, (T-PAMI), 1986. PDF.

The Canny Edge-Detection Algorithm (1986)

4. **Thinning**: look for local-maxima in the edge strength in the direction of the gradient



Thresholded $||\nabla I||$ $\rightarrow$

Canny, J., A Computational Approach To Edge Detection, IEEE Transactions on Pattern Analysis and Machine Intelligence, (T-PAMI), 1986. PDF.

The Canny Edge-Detection Algorithm (1986)

4. **Thinning**: look for local-maxima in the edge strength in the direction of the gradient

   - This can be done by taking the **directional derivative of the edge strength in the direction of the gradient** and then looking for **zero-crossing** (i.e., adjacent pixel locations where the sign changes value)

   - The desired directional derivative is mathematically equivalent to convolving the image $I(x, y)$ with the Laplacian of Gaussian

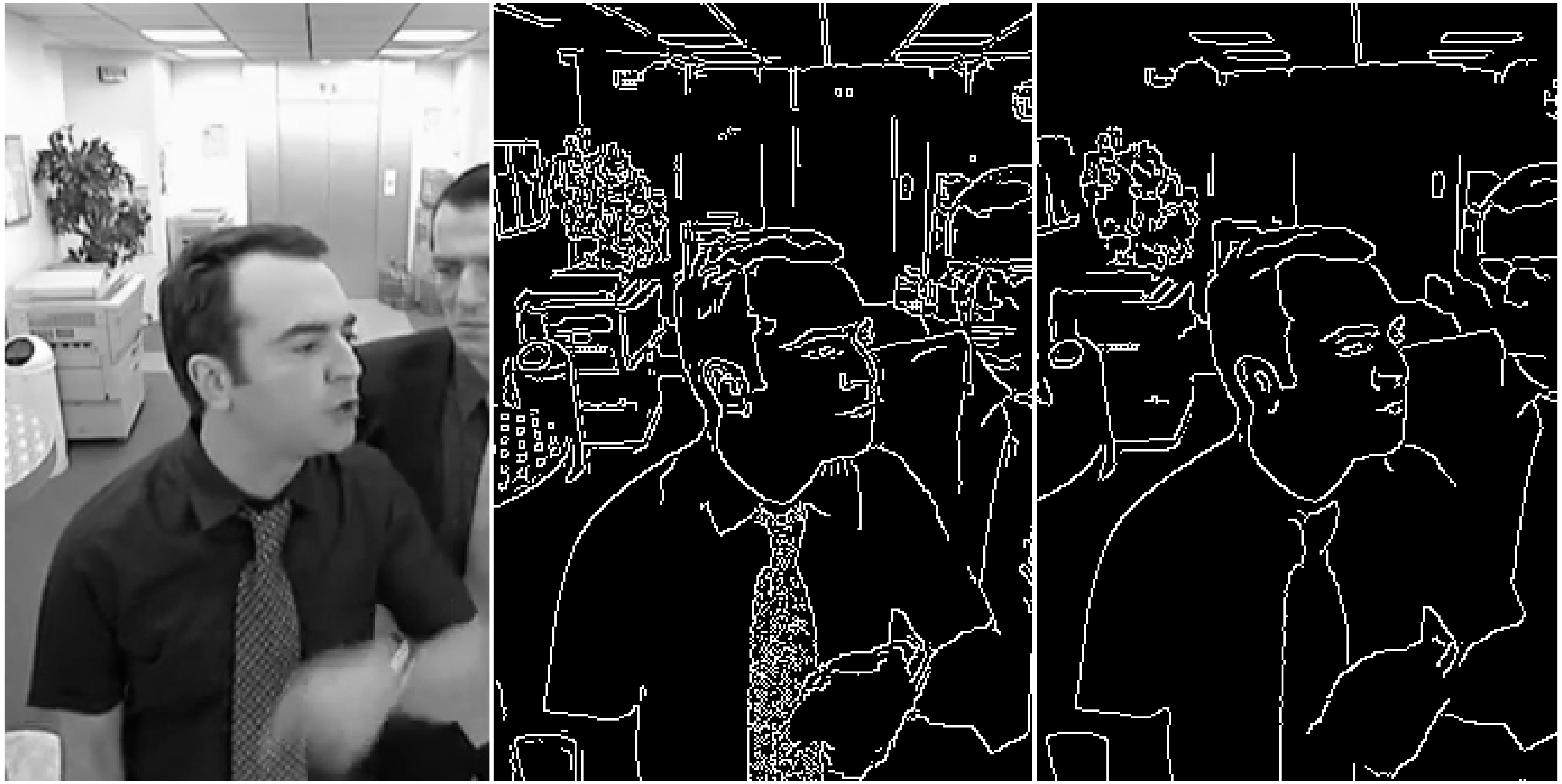
$$\nabla(\nabla G_\sigma * \mathrm{I}) = \nabla^2 G_\sigma * \mathrm{I}$$

Edge image: each pixel that is a local maximum of the edge strength in the direction of gradient is set to 1

Canny, J., A Computational Approach To Edge Detection, IEEE Transactions on Pattern Analysis and Machine Intelligence, (T-PAMI), 1986. PDF.

The Canny Edge-Detection Algorithm (1986)

What parameters can we tune to remove high frequency details?



Today: Deep Learning-based Edge Detection

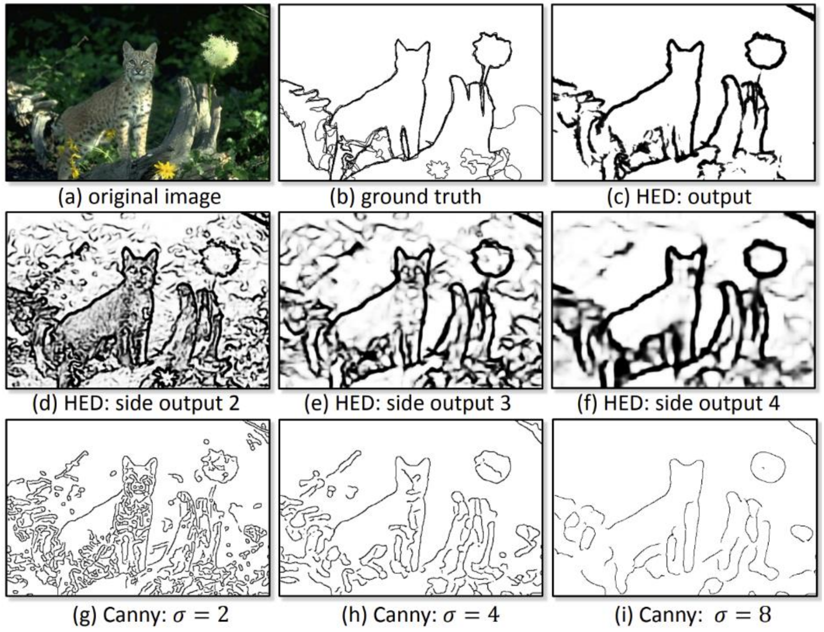
Supervised learning from human annotations

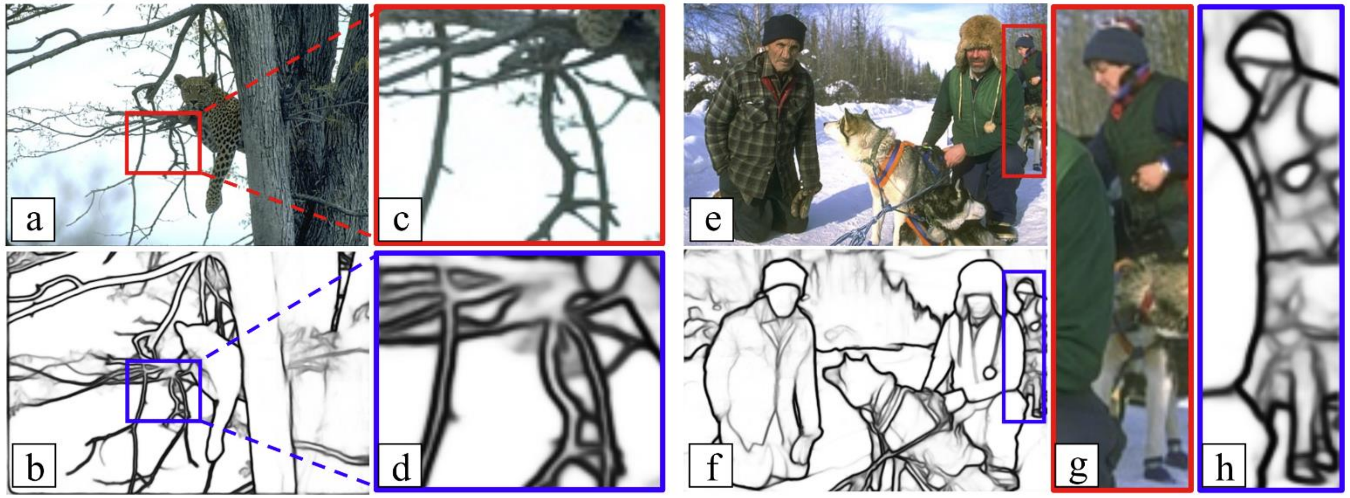
HED[1]**:** *CNN-based* Detector in 2015

- **>30% better** performance
- less computation than Canny

EDTER[2]**:** *State-of-the-art* approach

- Fine edges detection using *Transformer* model
- Integration with **global information**





[1] Xie et al., *Holistically-Nested Edge Detection*, International Conference on Computer Vision (ICCV), 2015. PDF.
[2] Pu et al., *EDTER: Edge Detection with Transformer*, Conference on Computer Vision and Pattern Recognition (CVPR), 2022. PDF.

Summary (things to remember)

- Image filtering (definition, motivation, applications)
- Moving average
- Linear filters and formulation: box filter, Gaussian filter
- Boundary issues
- Non-linear filters
- Median & bilateral filters
- Edge detection
- Derivating filters (Prewitt, Sobel)
- Combined derivative and smoothing filters (deriv. of Gaussian)
- Laplacian of Gaussian
- Canny edge detector

Readings

- Ch. 3.2, 3.3, 7.2.1 of Szeliski book, 2^{nd} Edition

Understanding Check

Are you able to:

- Explain the differences between convolution and cross-correlation?
- Explain the differences between a box filter and a Gaussian filter?
- Explain why one should increase the size of the kernel of a Gaussian filter if 2σ is close to the size of the kernel?
- Explain when we would need a median & bilateral filter?
- Explain how to handle boundary issues?
- Explain the working principle of edge detection with a $1\mathrm{D}$ signal?
- Explain how noise does affect this procedure?
- Explain the differential property of convolution?
- Show how to compute the first derivative of an image intensity function along x and y?
- Explain why the Laplacian of Gaussian operator is useful?
- List the properties of smoothing and derivative filters?
- Illustrate the Canny edge detection algorithm?
- Explain what non-maxima suppression is and how it is implemented?