Calibration by correlation using metric embedding from non-metric similarities

Appendix B: complete statistics and visualization

TABLE B-1: Observability case A: real **noisy** data

(a) Spearman score — equation 6

	g. truth	$\text{MDS}_{\mathbb{S}^n}$	SKv	SKv+w
FLIP/$\mathsf{corr}(y)$	0.9990	0.9699	0.9989	0.9996
GOPRO/$\mathsf{corr}(y)$	0.9949	0.9652	0.9975	0.9977
OMNI/$\mathsf{corr}(y)$	0.9173	0.9019	0.9438	0.9438

(b) Normalized Spearman score — equation 7

	g. truth	$\text{MDS}_{\mathbb{S}^n}$	SKv	SKv+w
FLIP/$\mathsf{corr}(y)$	1.0000	0.9709	0.9998	1.0006
GOPRO/$\mathsf{corr}(y)$	1.0000	0.9702	1.0027	1.0029
OMNI/$\mathsf{corr}(y)$	1.0000	0.9831	1.0288	1.0288

(c) Procrustes error — equation 3

	g. truth	$\text{MDS}_{\mathbb{S}^n}$	SKv	SKv+w
FLIP/$\mathsf{corr}(y)$	0.00	15.16	24.05	0.74
GOPRO/$\mathsf{corr}(y)$	0.00	6.20	4.72	3.53
OMNI/$\mathsf{corr}(y)$	0.02	32.43	9.48	9.48

(d) Relative error — equation 4

	g. truth	$\text{MDS}_{\mathbb{S}^n}$	SKv	SKv+w
FLIP/$\mathsf{corr}(y)$	0.00	7.14	11.87	0.36
GOPRO/$\mathsf{corr}(y)$	0.00	1.72	0.95	0.79
OMNI/$\mathsf{corr}(y)$	0.00	2.20	1.80	1.80

(e) Scaled relative error — equation 5

	g. truth	$\text{MDS}_{\mathbb{S}^n}$	SKv	SKv+w
FLIP/$\mathsf{corr}(y)$	0.00	1.79	0.40	0.33
GOPRO/$\mathsf{corr}(y)$	0.00	5.99	2.51	2.55
OMNI/$\mathsf{corr}(y)$	0.00	11.90	8.91	8.91

(f) Diameter — equation 1

	g. truth	$\text{MDS}_{\mathbb{S}^n}$	SKv	SKv+w
FLIP/$\mathsf{corr}(y)$	45.13	75.15	133.19	42.63
GOPRO/$\mathsf{corr}(y)$	150.99	111.82	129.79	153.06
OMNI/$\mathsf{corr}(y)$	329.54	221.53	312.77	312.77

TABLE B-2: Observability case A: synthetic **noiseless** data

(a) Spearman score — equation 6

	g. truth	$\text{MDS}_{\mathbb{S}^n}$	SKv	SKv+w
FLIP/f_{exp}	1.0000	0.9704	0.9992	1.0000
GOPRO/f_{exp}	1.0000	0.9499	0.9999	1.0000
OMNI/f_{exp}	1.0000	0.9739	1.0000	1.0000

(b) Normalized Spearman score — equation 7

	g. truth	$\text{MDS}_{\mathbb{S}^n}$	SKv	SKv+w
FLIP/f_{exp}	1.0000	0.9704	0.9992	1.0000
GOPRO/f_{exp}	1.0000	0.9499	0.9999	1.0000
OMNI/f_{exp}	1.0000	0.9739	1.0000	1.0000

(c) Procrustes error — equation 3

	g. truth	$\text{MDS}_{\mathbb{S}^n}$	SKv	SKv+w
FLIP/f_{exp}	0.00	16.17	24.07	1.25
GOPRO/f_{exp}	0.00	8.04	3.52	0.90
OMNI/f_{exp}	0.02	37.07	0.00	0.00

(d) Relative error — equation 4

	g. truth	$\text{MDS}_{\mathbb{S}^n}$	SKv	SKv+w
FLIP/f_{exp}	0.00	7.54	11.79	0.63
GOPRO/f_{exp}	0.00	2.00	0.55	0.12
OMNI/f_{exp}	0.00	2.95	0.00	0.00

(e) Scaled relative error — equation 5

	g. truth	$\text{MDS}_{\mathbb{S}^n}$	SKv	SKv+w
FLIP/f_{exp}	0.00	1.83	0.26	0.06
GOPRO/f_{exp}	0.00	7.60	0.26	0.12
OMNI/f_{exp}	0.00	9.54	0.00	0.00

(f) Diameter — equation 1

	g. truth	$\text{MDS}_{\mathbb{S}^n}$	SKv	SKv+w
FLIP/f_{exp}	45.13	78.76	134.75	41.13
GOPRO/f_{exp}	150.99	112.33	136.53	154.00
OMNI/f_{exp}	329.54	189.61	329.54	329.54

TABLE B-3: Observability case A: multiple statistics

(a) Spearman score — equation 6

	g. truth	SKv+w
OMNI/corr(y)	0.9173	0.9438
OMNI/corr$(c(y))$	0.9212	0.9465
OMNI/corr$(\dot{y})$	0.8550	0.9211
OMNI/corr$(\mathsf{sgn}(\dot{y}))$	0.8739	0.9077

(b) Normalized Spearman score — equation 7

	g. truth	SKv+w
OMNI/corr(y)	1.0000	1.0288
OMNI/corr$(c(y))$	1.0000	1.0274
OMNI/corr$(\dot{y})$	1.0000	1.0773
OMNI/corr$(\mathsf{sgn}(\dot{y}))$	1.0000	1.0387

(c) Procrustes error — equation 3

	g. truth	SKv+w
OMNI/corr(y)	0.02	9.48
OMNI/corr$(c(y))$	0.02	9.30
OMNI/corr$(\dot{y})$	0.02	16.07
OMNI/corr$(\mathsf{sgn}(\dot{y}))$	0.02	10.24

(d) Relative error — equation 4

	g. truth	SKv+w
OMNI/corr(y)	0.00	1.80
OMNI/corr$(c(y))$	0.00	1.75
OMNI/corr$(\dot{y})$	0.00	2.35
OMNI/corr$(\mathsf{sgn}(\dot{y}))$	0.00	2.61

(e) Scaled relative error — equation 5

	g. truth	SKv+w
OMNI/corr(y)	0.00	8.91
OMNI/corr$(c(y))$	0.00	8.72
OMNI/corr$(\dot{y})$	0.00	14.50
OMNI/corr$(\mathsf{sgn}(\dot{y}))$	0.00	9.22

(f) Diameter — equation 1

	g. truth	SKv+w
OMNI/corr(y)	329.54	312.77
OMNI/corr$(c(y))$	329.54	317.39
OMNI/corr$(\dot{y})$	329.54	284.65
OMNI/corr$(\mathsf{sgn}(\dot{y}))$	329.54	286.48

TABLE B-4: Benchmarks for observability case B (data in $\mathbb{S}^1$, observable scale)

(a) Spearman score — equation 6

	g. truth	$\text{MDS}_{\mathbb{S}^n}$	SKv	SKv+w
$\mathbb{S}^1$, 315deg, f_{steep}	0.8917	0.7739	0.8892	0.8892
$\mathbb{S}^1$, 315deg, f_{smooth}	0.9741	0.8357	0.9740	0.9740
$\mathbb{S}^1$, 315deg, f_{lin}	0.9993	0.8391	0.9993	0.9993

(b) Normalized Spearman score — equation 7

	g. truth	$\text{MDS}_{\mathbb{S}^n}$	SKv	SKv+w
$\mathbb{S}^1$, 315deg, f_{steep}	1.0000	0.8679	0.9972	0.9972
$\mathbb{S}^1$, 315deg, f_{smooth}	1.0000	0.8579	0.9999	0.9999
$\mathbb{S}^1$, 315deg, f_{lin}	1.0000	0.8397	1.0000	1.0000

(c) Procrustes error — equation 3

	g. truth	$\text{MDS}_{\mathbb{S}^n}$	SKv	SKv+w
$\mathbb{S}^1$, 315deg, f_{steep}	0.00	31.24	3.84	3.84
$\mathbb{S}^1$, 315deg, f_{smooth}	0.00	33.46	0.78	0.78
$\mathbb{S}^1$, 315deg, f_{lin}	0.00	40.26	0.07	0.07

(d) Relative error — equation 4

	g. truth	$\text{MDS}_{\mathbb{S}^n}$	SKv	SKv+w
$\mathbb{S}^1$, 315deg, f_{steep}	0.00	2.56	1.58	1.58
$\mathbb{S}^1$, 315deg, f_{smooth}	0.00	2.71	0.56	0.56
$\mathbb{S}^1$, 315deg, f_{lin}	0.00	2.85	0.09	0.09

(e) Scaled relative error — equation 5

	g. truth	$\text{MDS}_{\mathbb{S}^n}$	SKv	SKv+w
$\mathbb{S}^1$, 315deg, f_{steep}	0.00	26.98	5.17	5.17
$\mathbb{S}^1$, 315deg, f_{smooth}	0.00	26.65	1.07	1.07
$\mathbb{S}^1$, 315deg, f_{lin}	0.00	27.63	0.10	0.10

(f) Diameter — equation 1

	g. truth	$\text{MDS}_{\mathbb{S}^n}$	SKv	SKv+w
$\mathbb{S}^1$, 315deg, f_{steep}	317.92	126.81	312.15	312.15
$\mathbb{S}^1$, 315deg, f_{smooth}	317.92	114.04	316.75	316.75
$\mathbb{S}^1$, 315deg, f_{lin}	317.92	95.33	318.10	318.10

(g) Angular correlation

	g. truth	$\text{MDS}_{\mathbb{S}^n}$	SKv	SKv+w
$\mathbb{S}^1$, 315deg, f_{steep}	1.0000	0.9646	0.9991	0.9991
$\mathbb{S}^1$, 315deg, f_{smooth}	1.0000	0.9620	1.0000	1.0000
$\mathbb{S}^1$, 315deg, f_{lin}	1.0000	0.9417	1.0000	1.0000

TABLE B-5: Benchmarks for observability case C (data in $\mathbb{S}^1$, **unobservable** scale)

(a) Spearman score — equation 6

	g. truth	$\text{MDS}_{\mathbb{S}^n}$	SKv
$\mathbb{S}^1$, 90deg, f_{lin}	0.9999	0.9996	0.9999
$\mathbb{S}^1$, 45deg, f_{lin}	0.9999	0.9999	0.9999
$\mathbb{S}^1$, 45deg, f_{smooth}	0.9999	0.9987	0.9999
$\mathbb{S}^1$, 90deg, f_{smooth}	0.9999	0.9853	0.9997
FLIP (center)/$\text{corr}(y)$	0.9997	0.9706	0.9999
GOPRO (center)/$\text{corr}(y)$	0.9996	0.9592	0.9988

(b) Normalized Spearman score — equation 7

	g. truth	$\text{MDS}_{\mathbb{S}^n}$	SKv
$\mathbb{S}^1$, 90deg, f_{lin}	1.0000	0.9997	1.0000
$\mathbb{S}^1$, 45deg, f_{lin}	1.0000	1.0000	1.0000
$\mathbb{S}^1$, 45deg, f_{smooth}	1.0000	0.9988	1.0000
$\mathbb{S}^1$, 90deg, f_{smooth}	1.0000	0.9854	0.9998
FLIP (center)/$\text{corr}(y)$	1.0000	0.9708	1.0002
GOPRO (center)/$\text{corr}(y)$	1.0000	0.9596	0.9992

(c) Procrustes error — equation 3

	g. truth	$\text{MDS}_{\mathbb{S}^n}$	SKv
$\mathbb{S}^1$, 90deg, f_{lin}	0.00	7.27	8.43
$\mathbb{S}^1$, 45deg, f_{lin}	0.00	3.47	20.08
$\mathbb{S}^1$, 45deg, f_{smooth}	0.00	7.47	20.08
$\mathbb{S}^1$, 90deg, f_{smooth}	0.00	9.22	8.41
FLIP (center)/$\text{corr}(y)$	0.00	13.52	25.62
GOPRO (center)/$\text{corr}(y)$	0.00	8.58	13.01

(d) Relative error — equation 4

	g. truth	$\text{MDS}_{\mathbb{S}^n}$	SKv
$\mathbb{S}^1$, 90deg, f_{lin}	0.00	1.56	1.69
$\mathbb{S}^1$, 45deg, f_{lin}	0.00	1.42	8.07
$\mathbb{S}^1$, 45deg, f_{smooth}	0.00	2.79	8.07
$\mathbb{S}^1$, 90deg, f_{smooth}	0.00	1.69	1.62
FLIP (center)/$\text{corr}(y)$	0.00	9.49	19.88
GOPRO (center)/$\text{corr}(y)$	0.00	2.93	3.50

(e) Scaled relative error — equation 5

	g. truth	$\text{MDS}_{\mathbb{S}^n}$	SKv
$\mathbb{S}^1$, 90deg, f_{lin}	0.00	0.45	0.12
$\mathbb{S}^1$, 45deg, f_{lin}	0.00	0.07	0.06
$\mathbb{S}^1$, 45deg, f_{smooth}	0.00	0.43	0.06
$\mathbb{S}^1$, 90deg, f_{smooth}	0.00	3.19	0.30
FLIP (center)/$\text{corr}(y)$	0.00	1.13	0.11
GOPRO (center)/$\text{corr}(y)$	0.00	4.30	0.64

(f) Diameter — equation 1

	g. truth	$\text{MDS}_{\mathbb{S}^n}$	SKv
$\mathbb{S}^1$, 90deg, f_{lin}	89.76	60.27	122.05
$\mathbb{S}^1$, 45deg, f_{lin}	44.88	31.42	122.05
$\mathbb{S}^1$, 45deg, f_{smooth}	44.88	70.33	122.05
$\mathbb{S}^1$, 90deg, f_{smooth}	89.76	106.25	119.91
FLIP (center)/$\text{corr}(y)$	23.12	58.04	125.50
GOPRO (center)/$\text{corr}(y)$	73.41	78.51	128.97

(g) Angular correlation

	g. truth	$\text{MDS}_{\mathbb{S}^n}$	SKv
$\mathbb{S}^1$, 90deg, f_{lin}	1.0000	0.9985	0.9976
$\mathbb{S}^1$, 45deg, f_{lin}	1.0000	1.0000	1.0000
$\mathbb{S}^1$, 45deg, f_{smooth}	1.0000	0.9996	1.0000
$\mathbb{S}^1$, 90deg, f_{smooth}	1.0000	0.9971	0.9976
FLIP (center)/$\text{corr}(y)$	nan	0.9894	0.9999
GOPRO (center)/$\text{corr}(y)$	nan	0.9847	0.9996

TABLE B-6: Benchmarks for observability case D (Euclidean case, unobservable scale)

(a) Spearman score — equation 6

	g. truth	$\mathsf{MDS}_{\mathbb{R}^n}$	SKv
$\mathbb{R}^2$, f_{lin}	0.9998	0.9998	0.9998
$\mathbb{R}^2$, f_{steep}	0.9998	0.7704	0.9998
$\mathbb{R}^2$, f_{smooth}	0.9998	0.9180	0.9998

(b) Normalized Spearman score — equation 7

	g. truth	$\mathsf{MDS}_{\mathbb{R}^n}$	SKv
$\mathbb{R}^2$, f_{lin}	1.0000	1.0000	1.0000
$\mathbb{R}^2$, f_{steep}	1.0000	0.7706	1.0000
$\mathbb{R}^2$, f_{smooth}	1.0000	0.9182	1.0000

(c) Procrustes error — equation 3

	g. truth	$\mathsf{MDS}_{\mathbb{R}^n}$	SKv
$\mathbb{R}^2$, f_{lin}	0.00	0.46	0.03
$\mathbb{R}^2$, f_{steep}	0.00	0.28	0.03
$\mathbb{R}^2$, f_{smooth}	0.00	0.22	0.03

(d) Relative error — equation 4

	g. truth	$\mathsf{MDS}_{\mathbb{R}^n}$	SKv
$\mathbb{R}^2$, f_{lin}	0.0000	0.6295	0.0411
$\mathbb{R}^2$, f_{steep}	0.0000	0.3859	0.0411
$\mathbb{R}^2$, f_{smooth}	0.0000	0.3255	0.0411

(e) Scaled relative error — equation 5

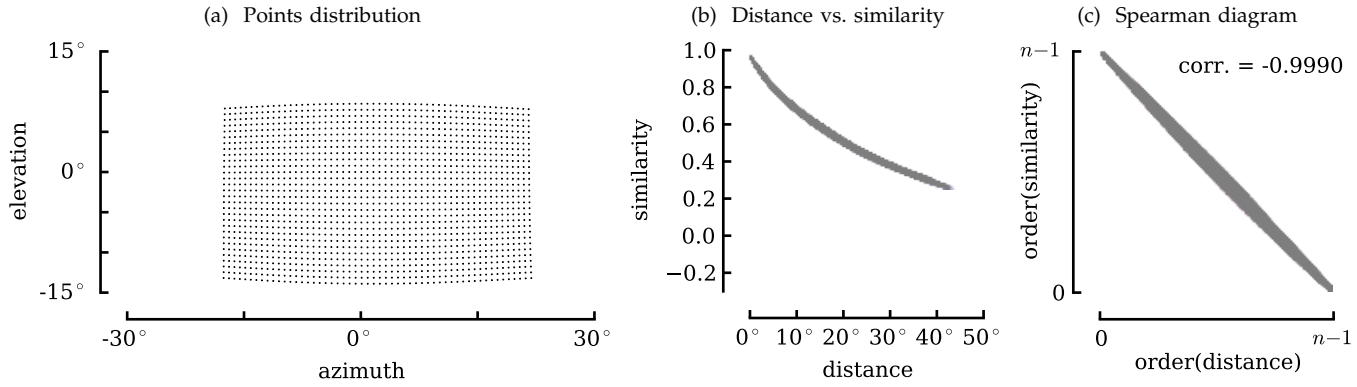
	g. truth	$\mathsf{MDS}_{\mathbb{R}^n}$	SKv
$\mathbb{R}^2$, f_{lin}	0.000	0.003	0.003
$\mathbb{R}^2$, f_{steep}	0.000	0.237	0.003
$\mathbb{R}^2$, f_{smooth}	0.000	0.153	0.003

(f) Diameter — equation 1

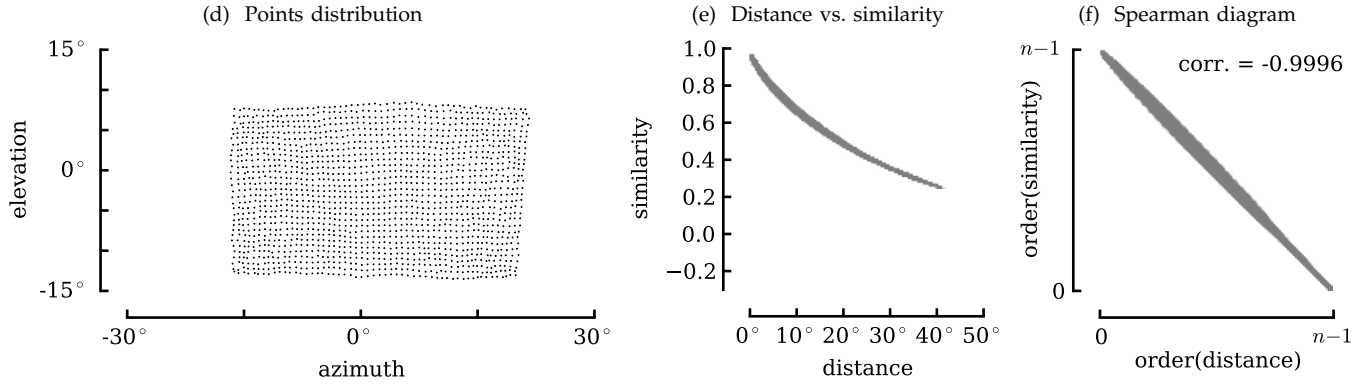
	g. truth	$\mathsf{MDS}_{\mathbb{R}^n}$	SKv
$\mathbb{R}^2$, f_{lin}	2.5302	0.8010	2.6160
$\mathbb{R}^2$, f_{steep}	2.5302	1.1413	2.6160
$\mathbb{R}^2$, f_{smooth}	2.5302	1.1590	2.6160

Fig. B-1: FLIP/corr(y)

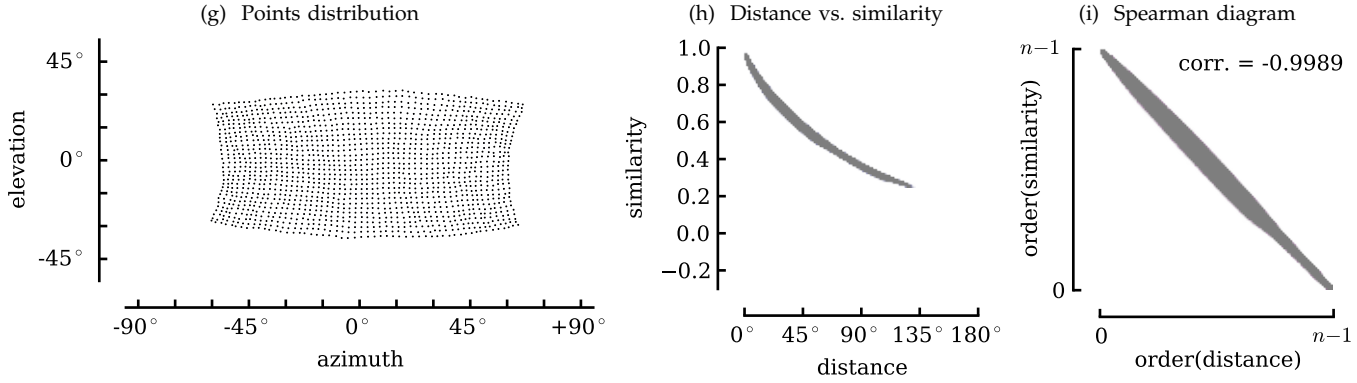
Ground truth



SKv+w



SKv



$\text{MDS}_{\mathbb{S}^n}$

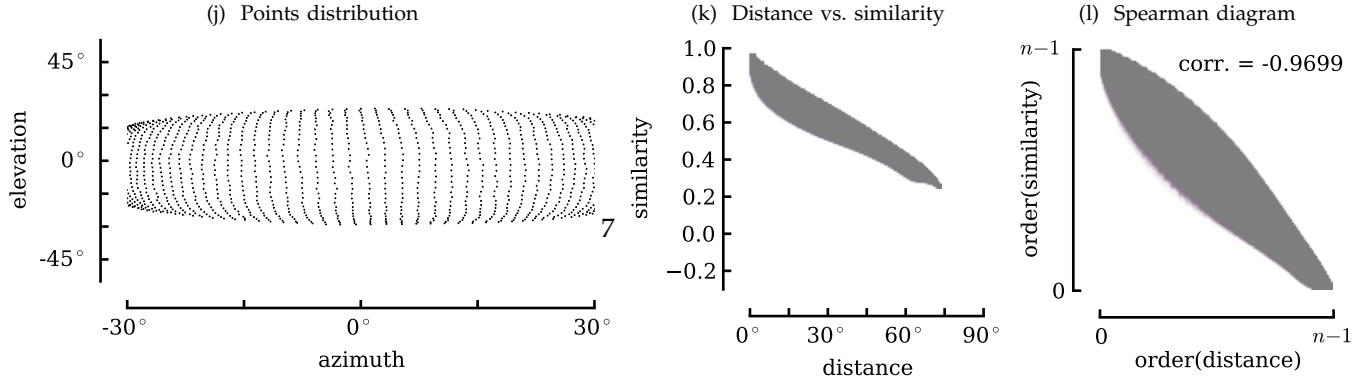
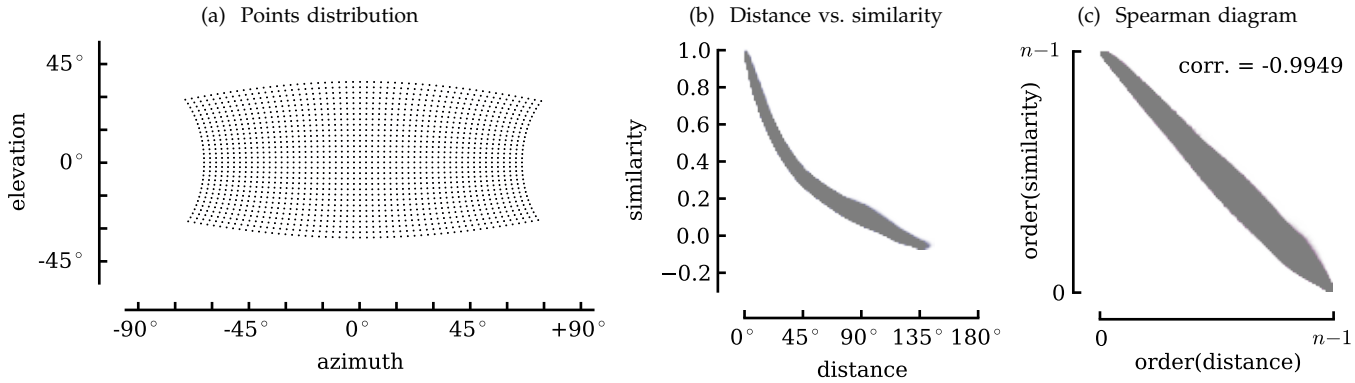
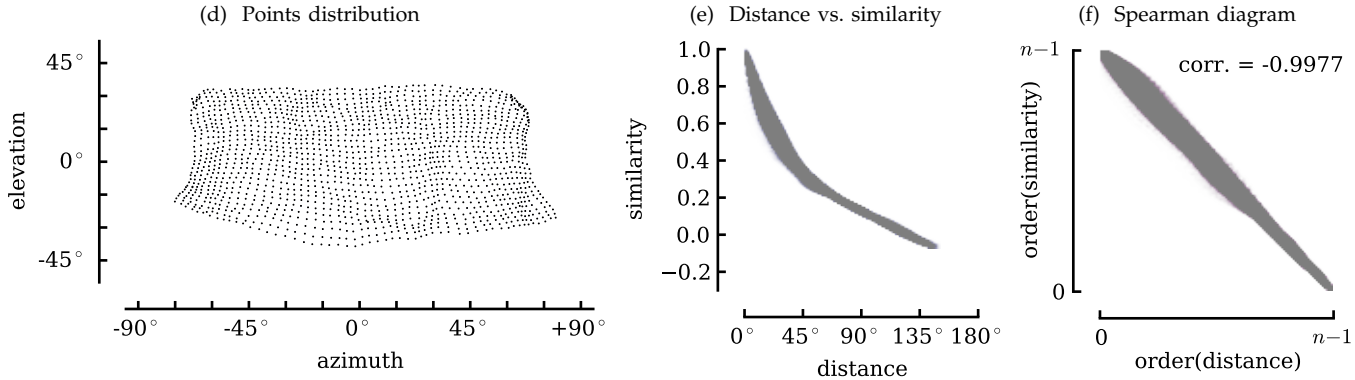


Fig. B-2: GOPRO/corr(y)

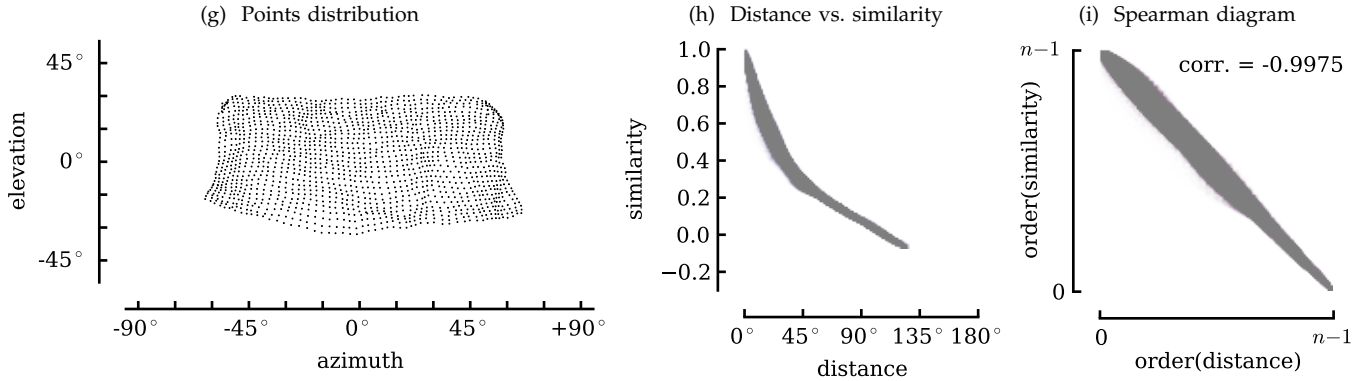
Ground truth



SKv+w



SKv



$\mathsf{MDS}_{\mathbb{S}^n}$

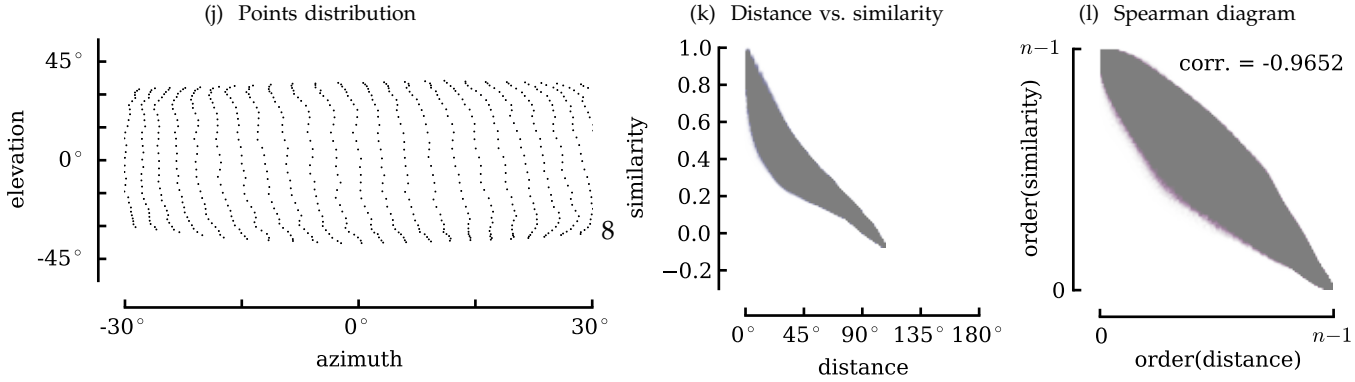
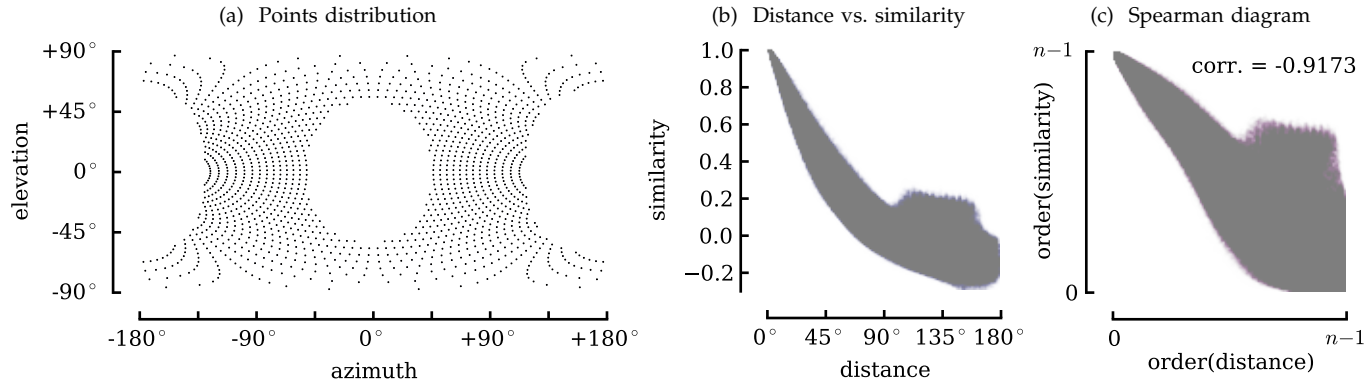
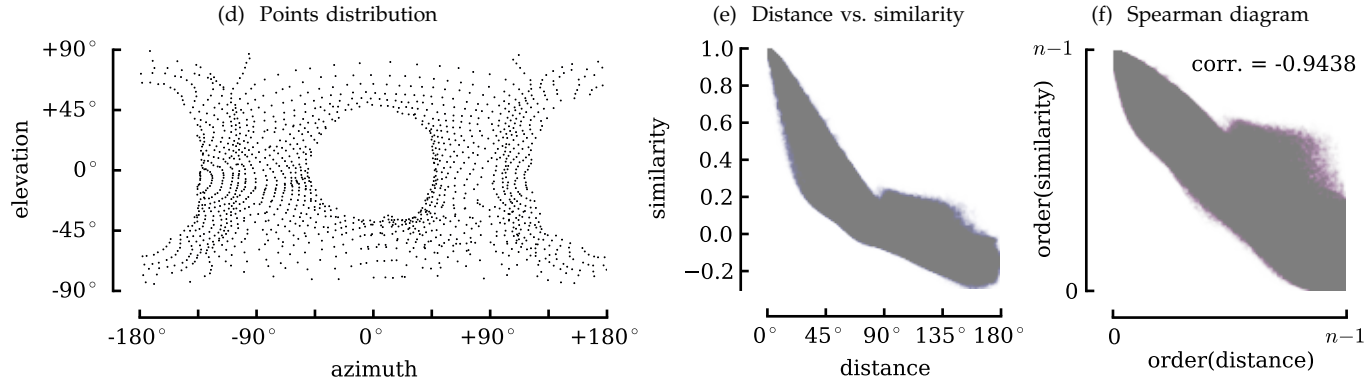


Fig. B-3: OMNI/corr(y)

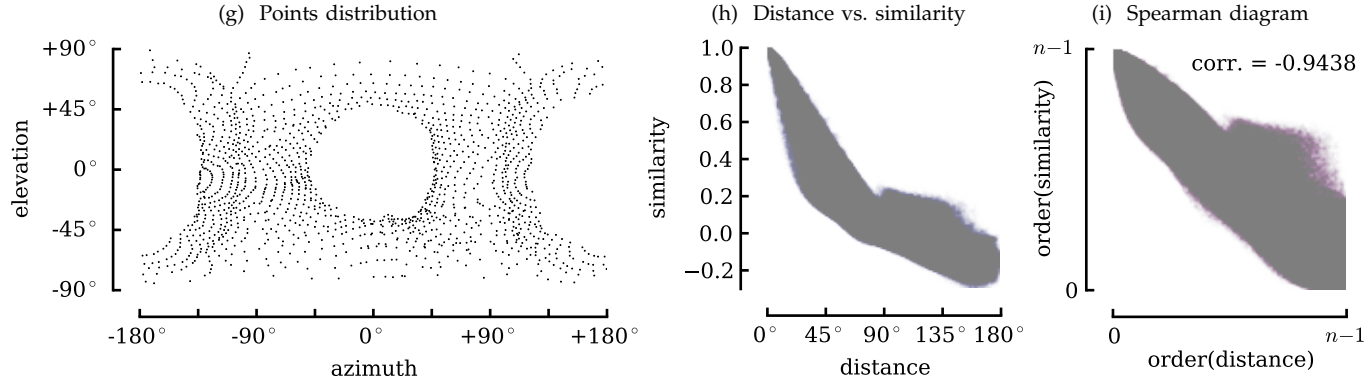
Ground truth



SKv+w



SKv



$\text{MDS}_{\mathbb{S}^n}$

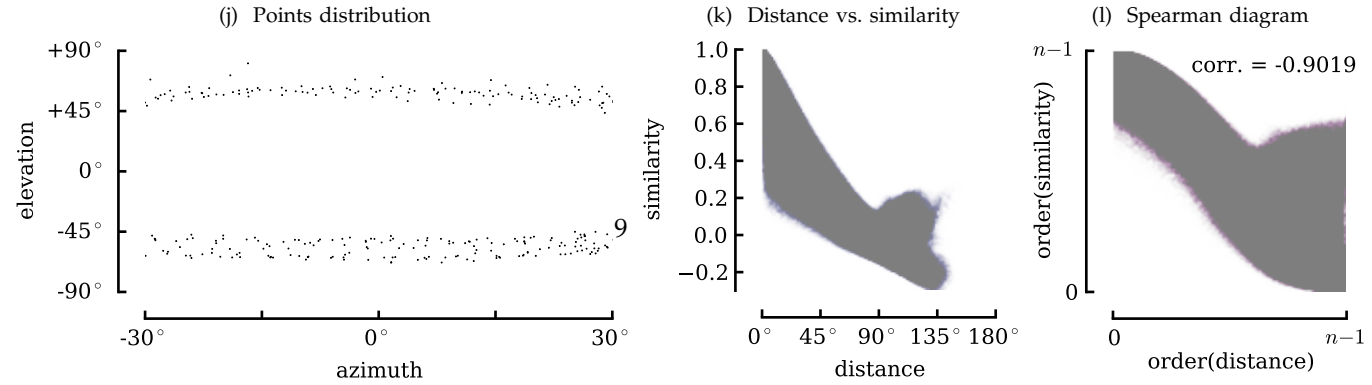
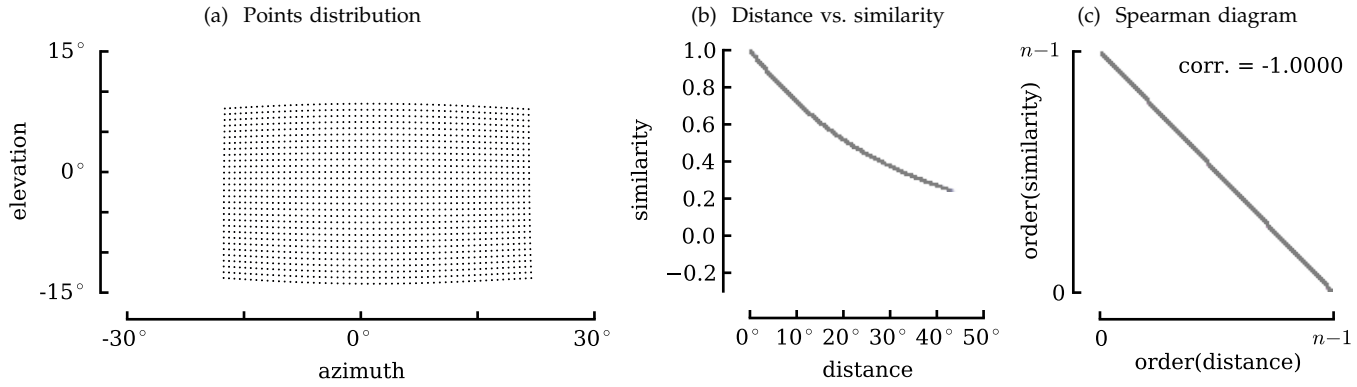
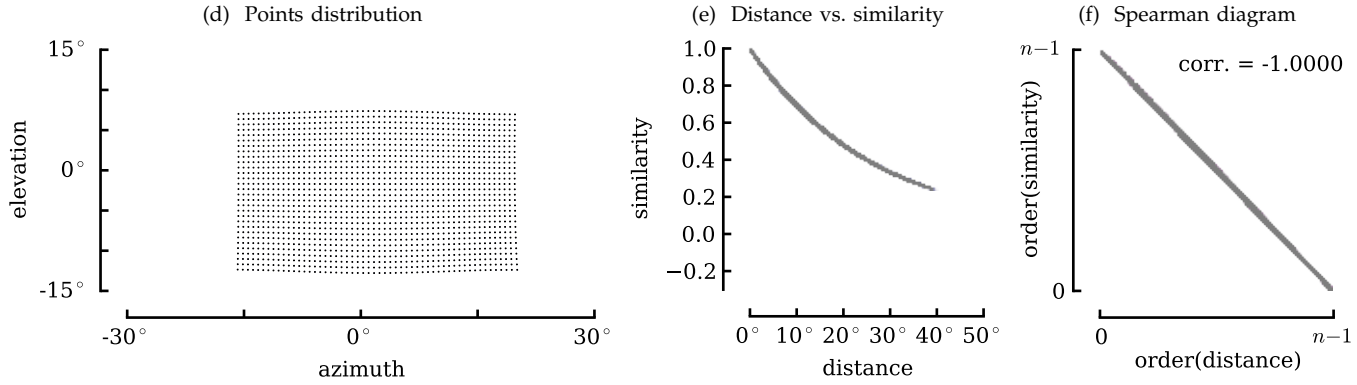


Fig. B-4: FLIP / f_{exp}

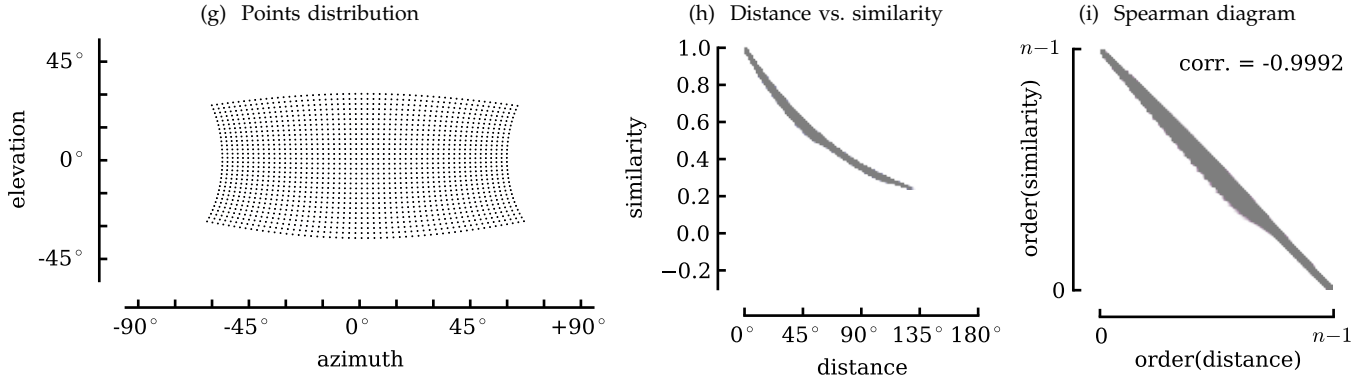
Ground truth



SKv+w



SKv



$\text{MDS}_{\mathbb{S}^n}$

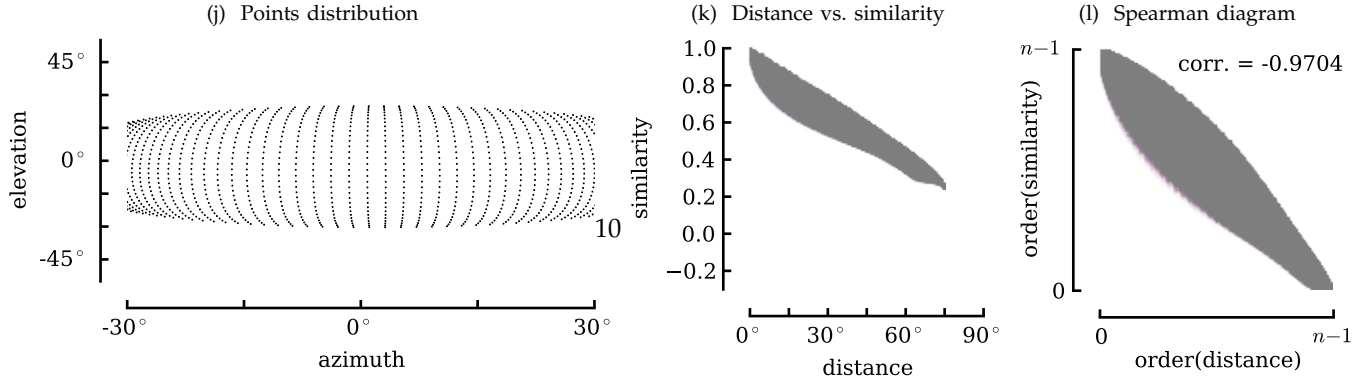
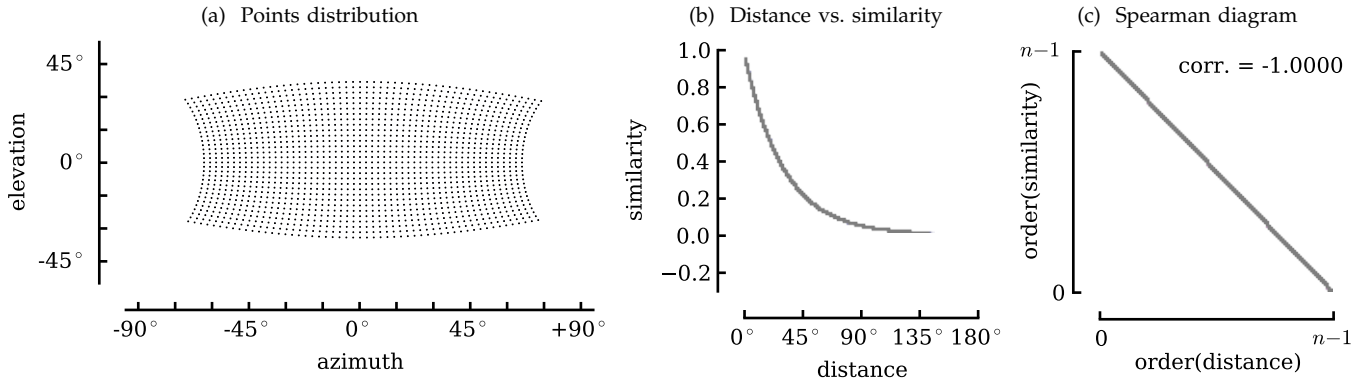
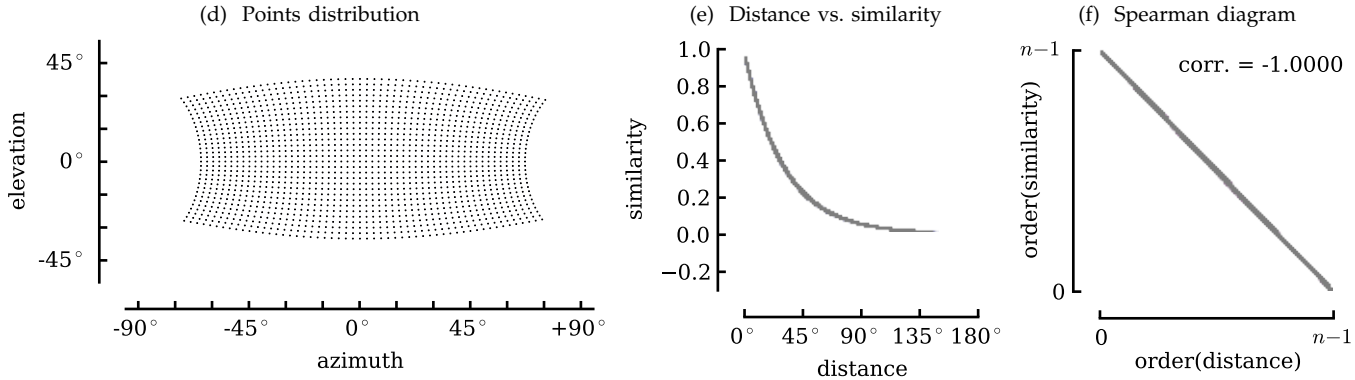


Fig. B-5: GOPRO/f_{exp}

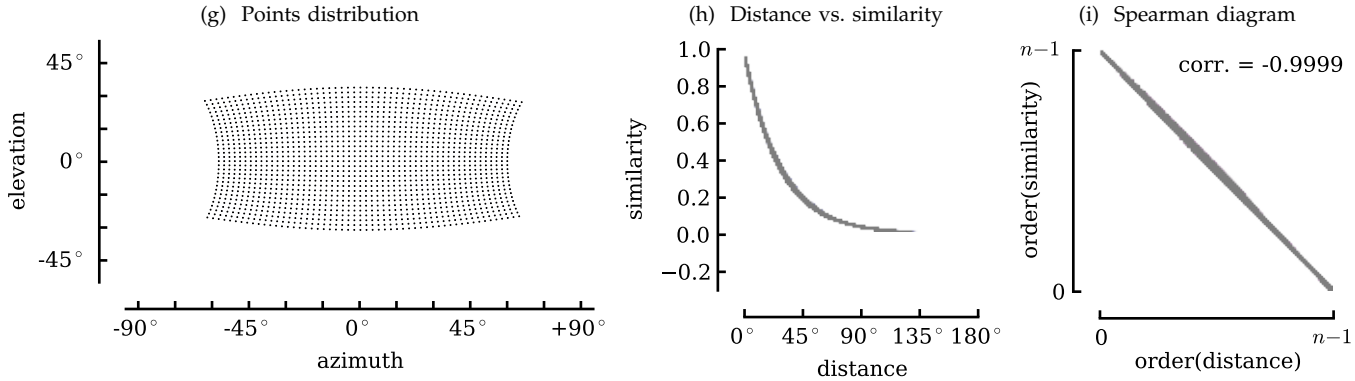
Ground truth



SKv+w



SKv



$\text{MDS}_{\mathbb{S}^n}$

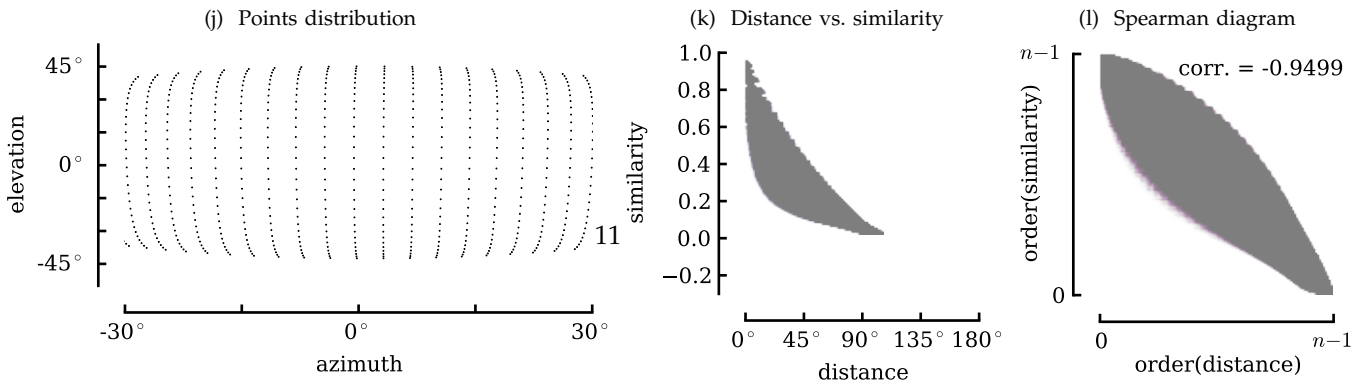
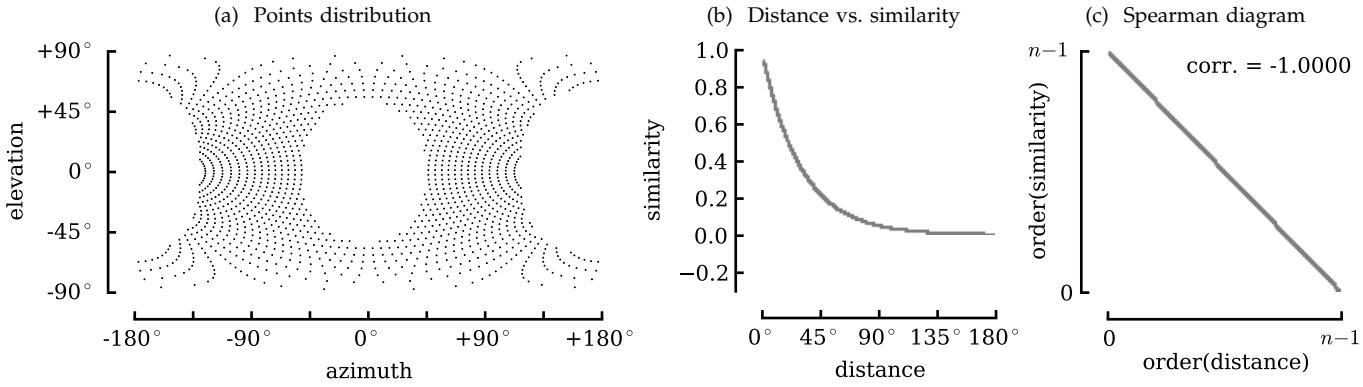
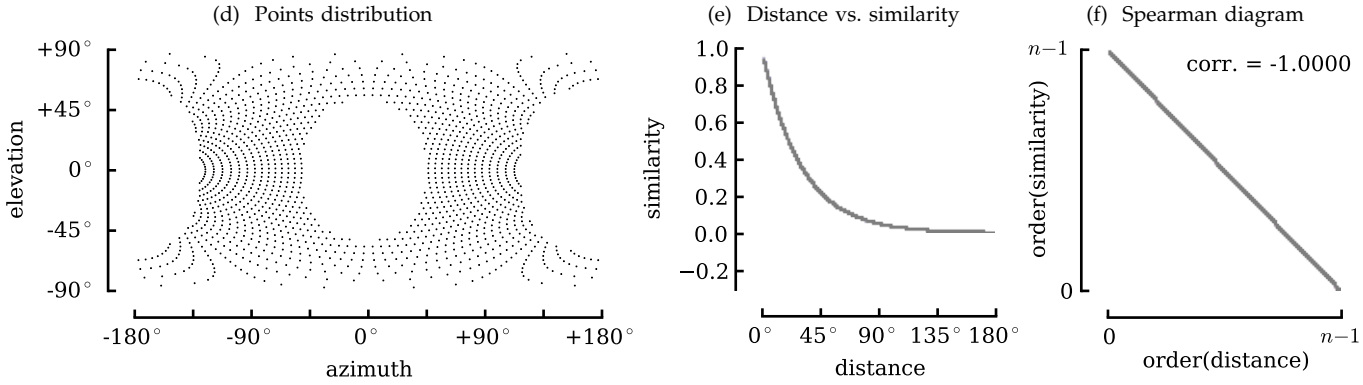


Fig. B-6: OMNI/f_{exp}

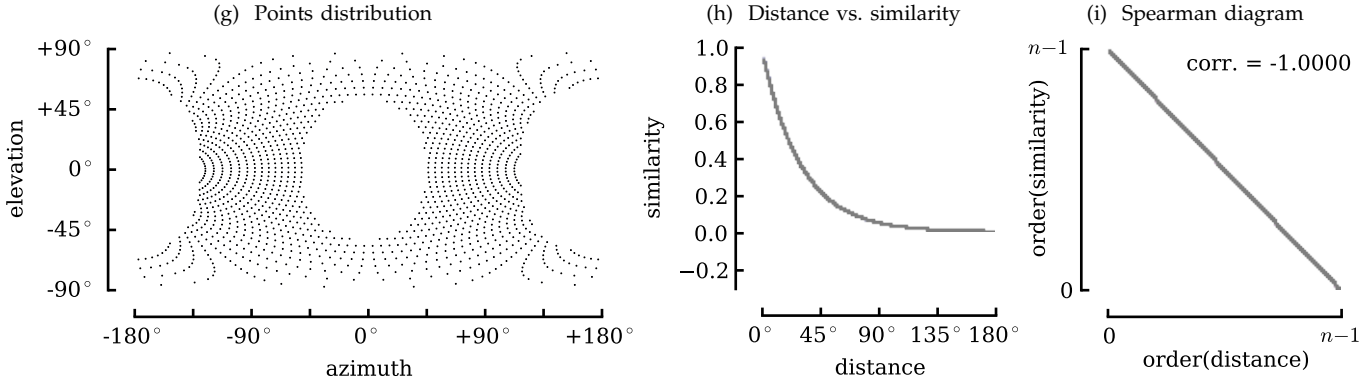
Ground truth



SKv+w



SKv



$\text{MDS}_{\mathbb{S}^n}$

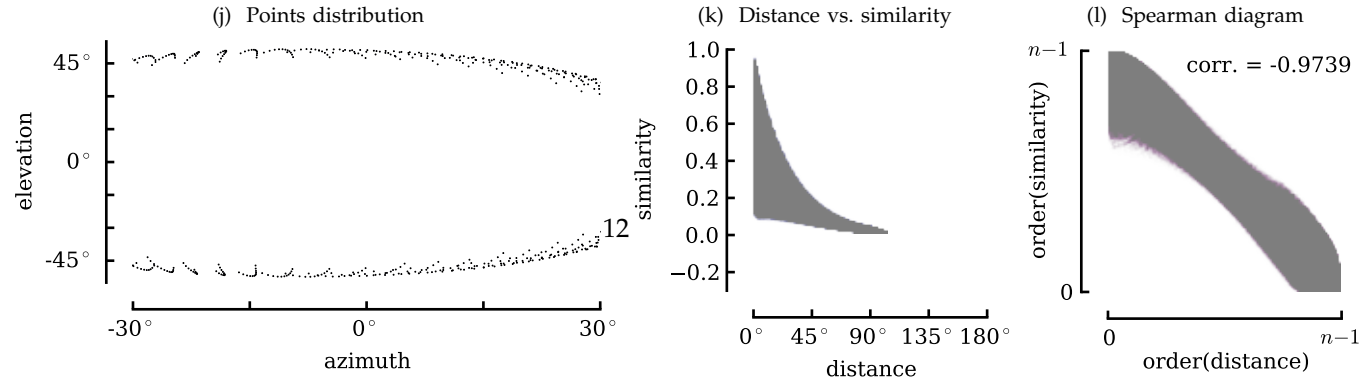
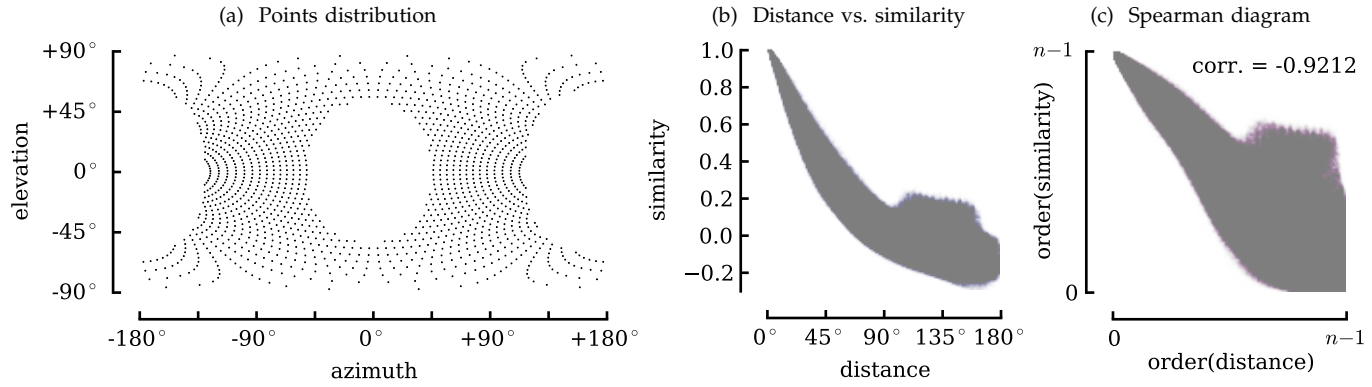


Fig. B-7: OMNI/corr$(c(y))$

Ground truth



SKv+w

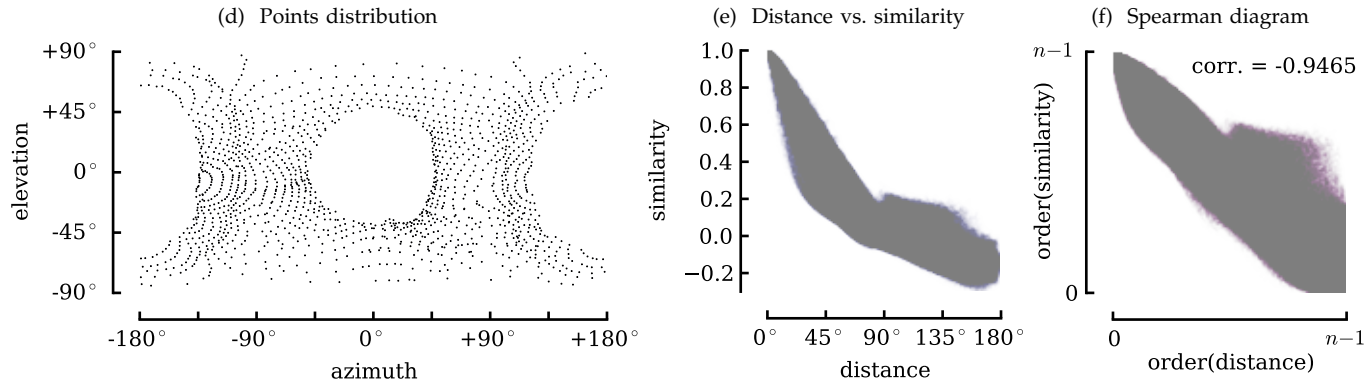
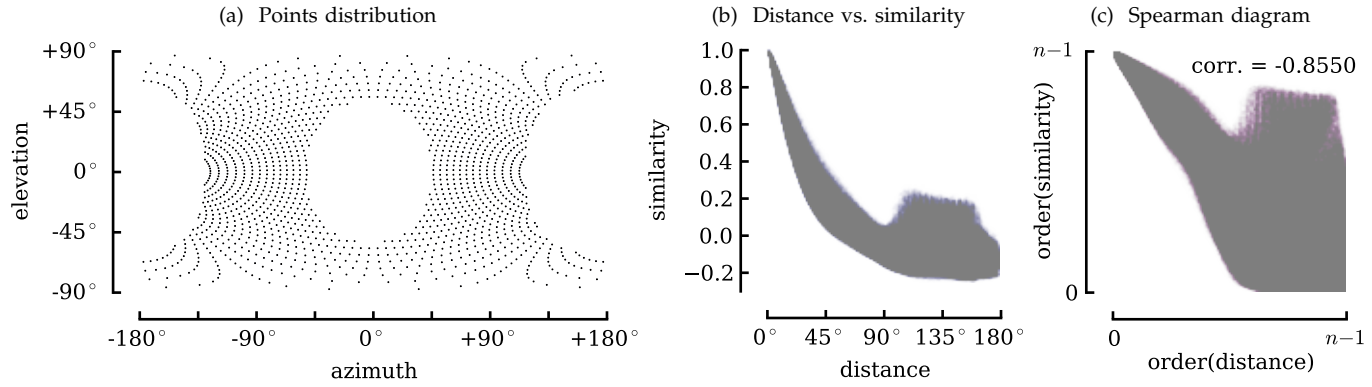


Fig. B-8: OMNI/corr($\dot{y}$))

Ground truth



SKv+w

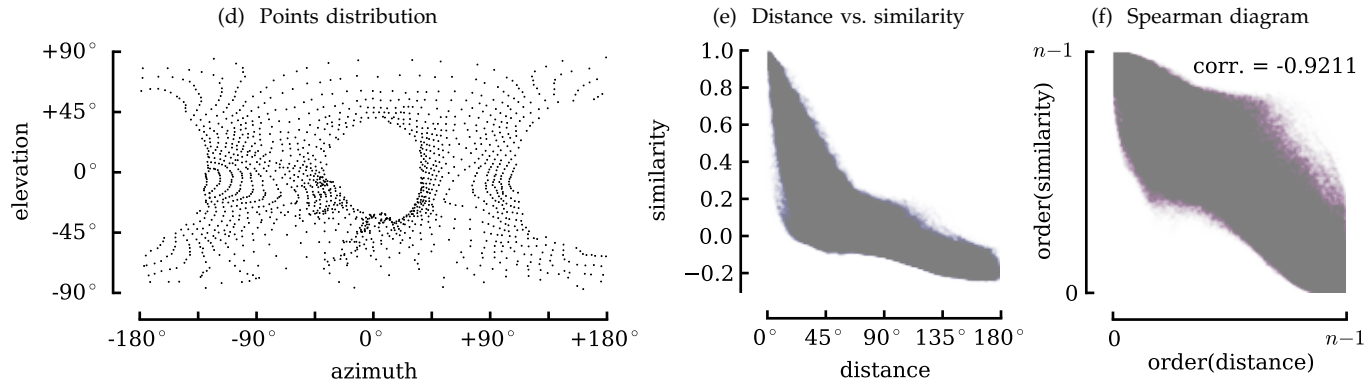
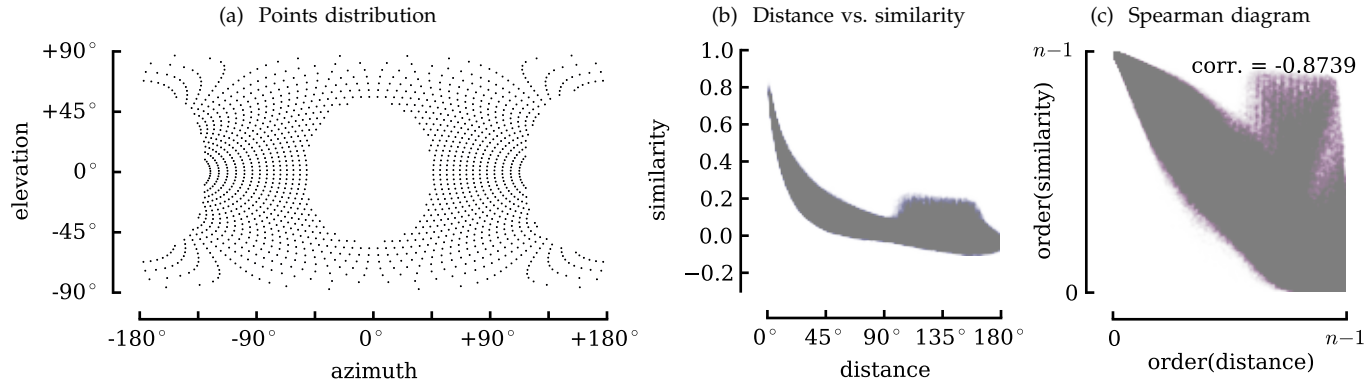


Fig. B-9: OMNI/corr(sgn($\dot{y}$))

Ground truth



SKv+w

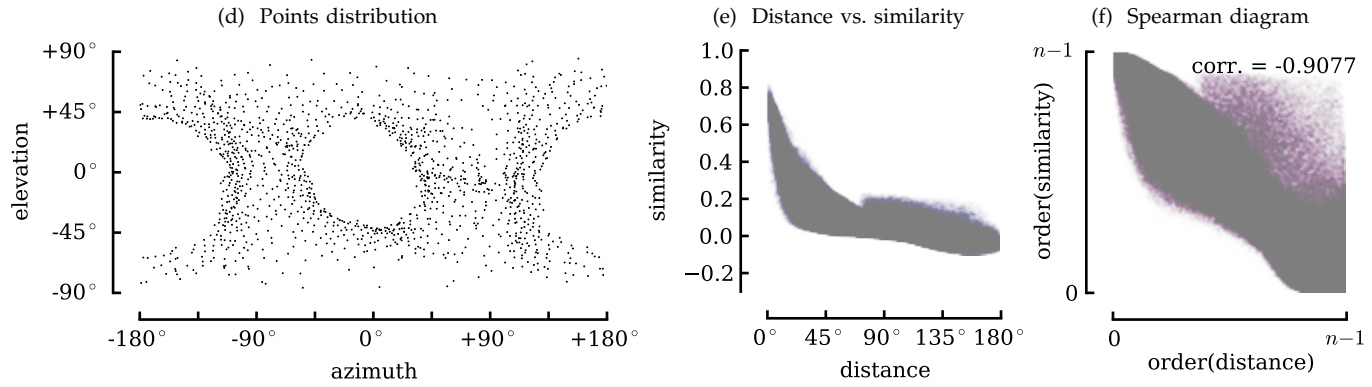
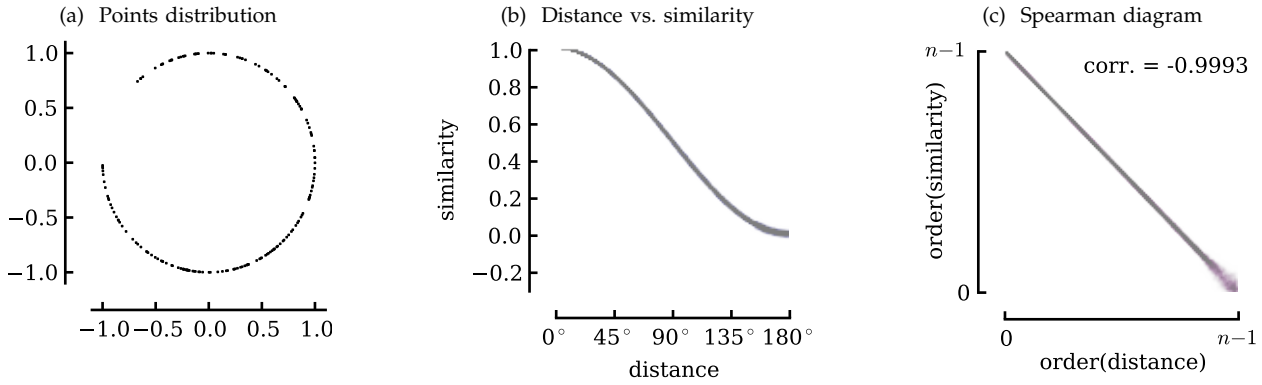
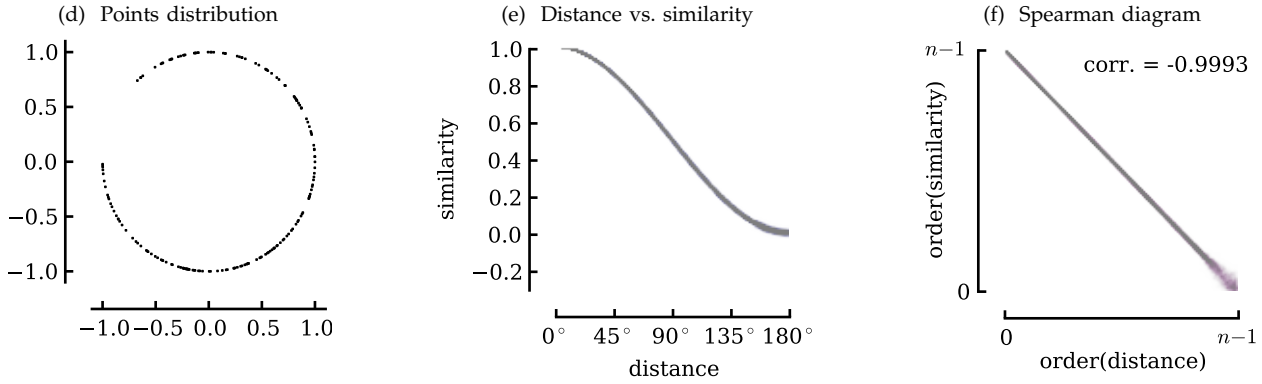


Fig. B-10: Random distribution on $\mathbb{S}^2$, 315deg fov, f_{lin}

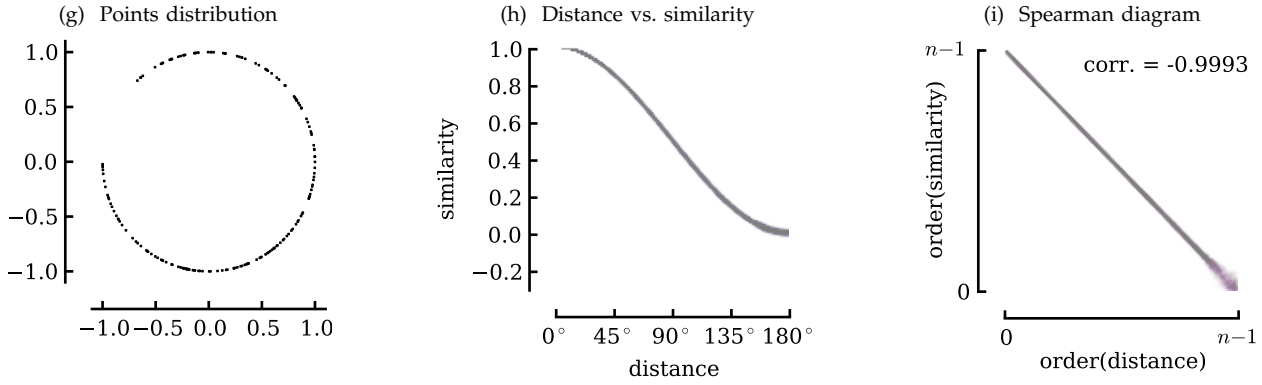
Ground truth



SKv+w



SKv



$\text{MDS}_{\mathbb{S}^n}$

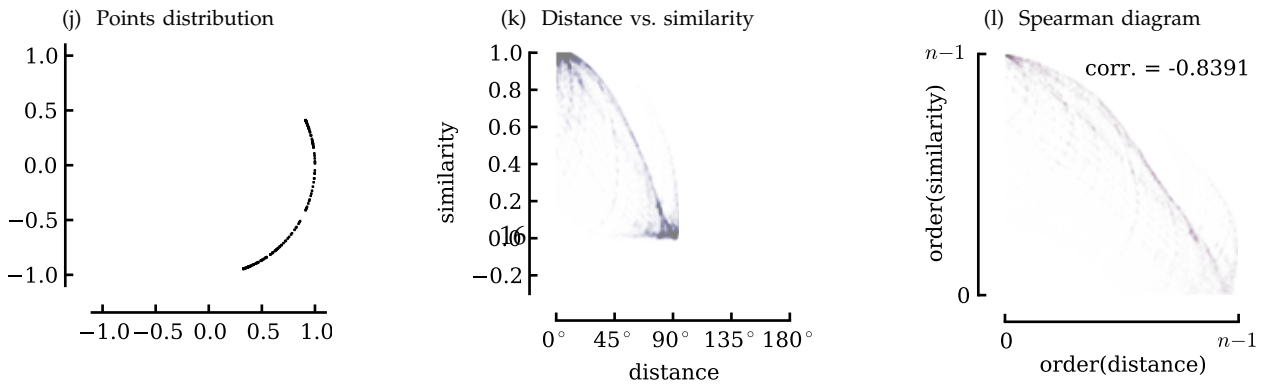
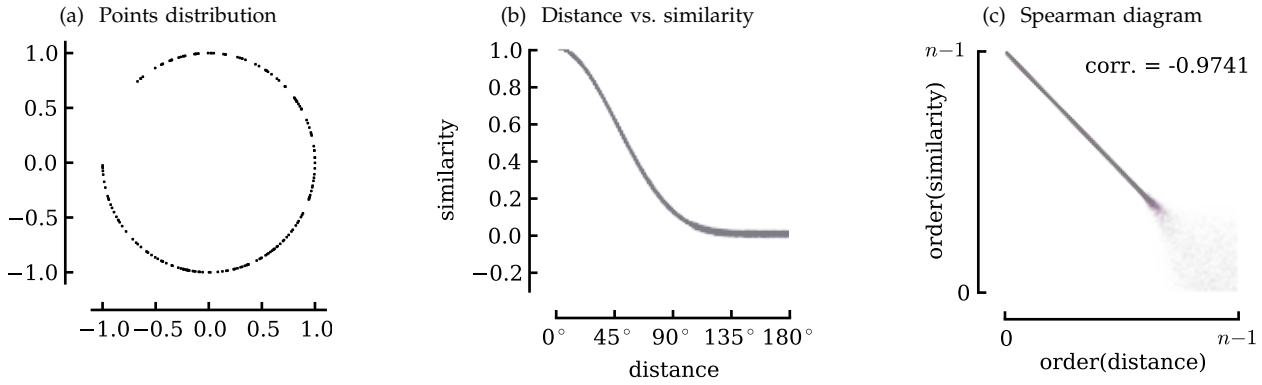
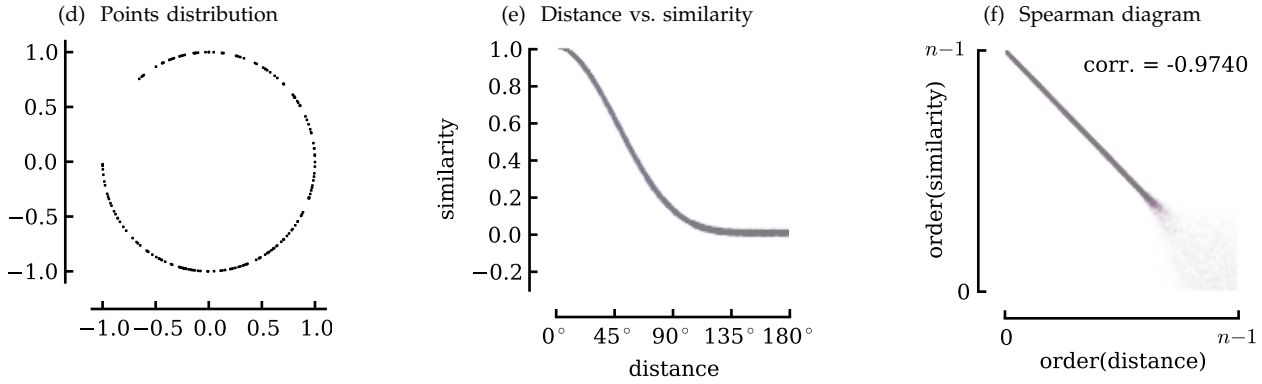


Fig. B-11: Random distribution on $\mathbb{S}^2$, 315deg fov, f_{smooth}

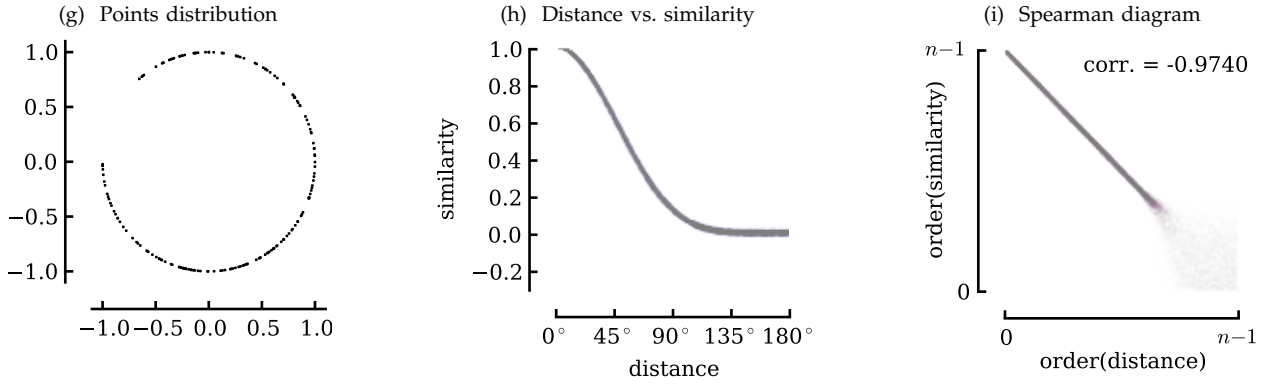
Ground truth



SKv+w



SKv



$\mathsf{MDS}_{\mathbb{S}^n}$

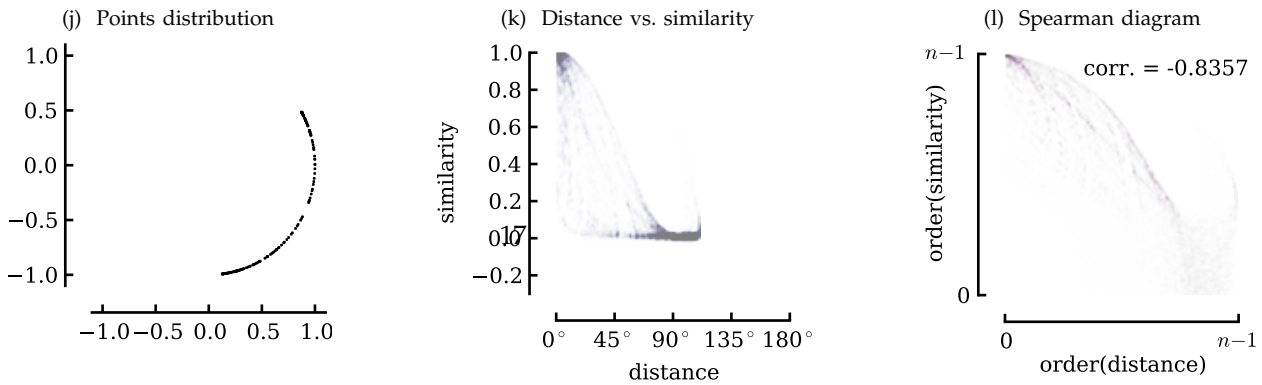
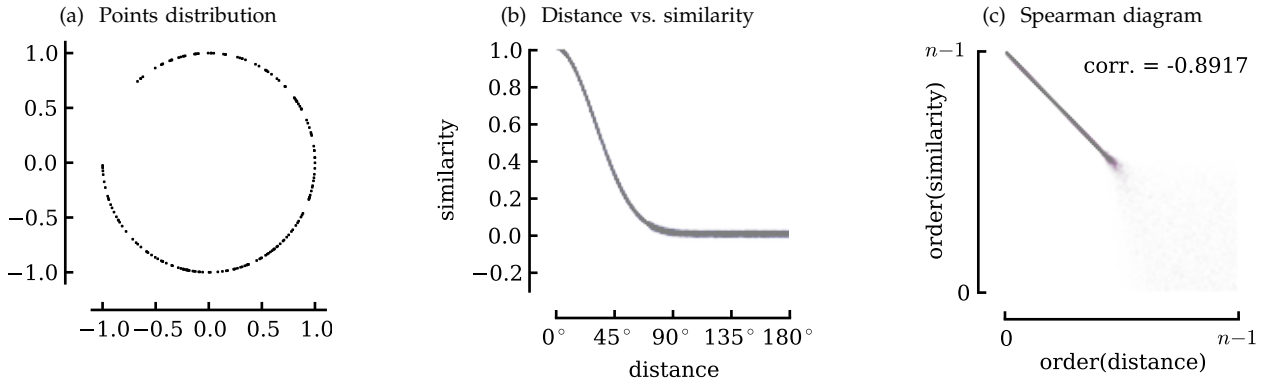
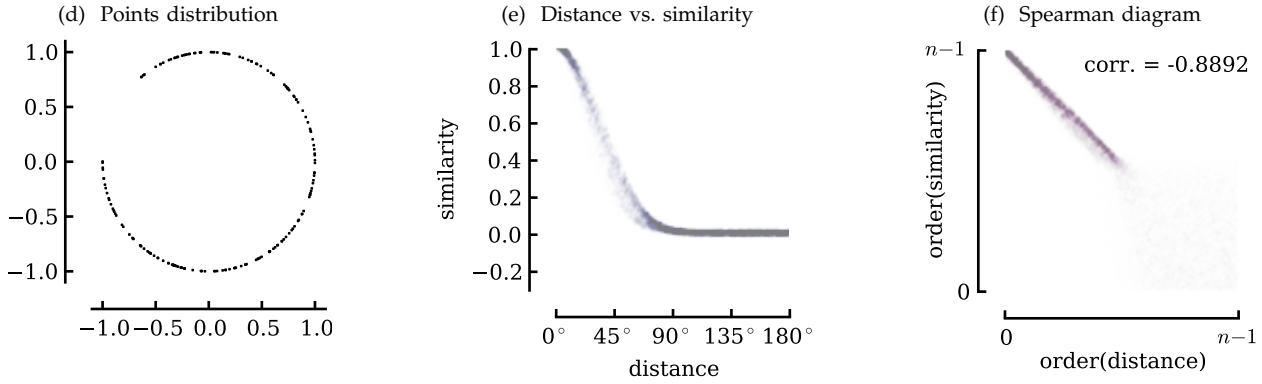


Fig. B-12: Random distribution on $\mathbb{S}^2$, 315deg fov, f_{steep}

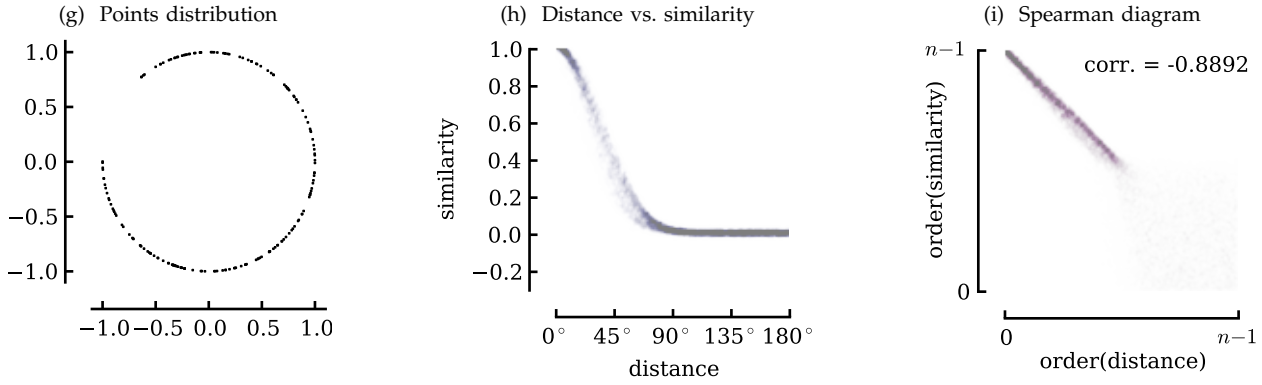
Ground truth



SKv+w



SKv



$\text{MDS}_{\mathbb{S}^n}$

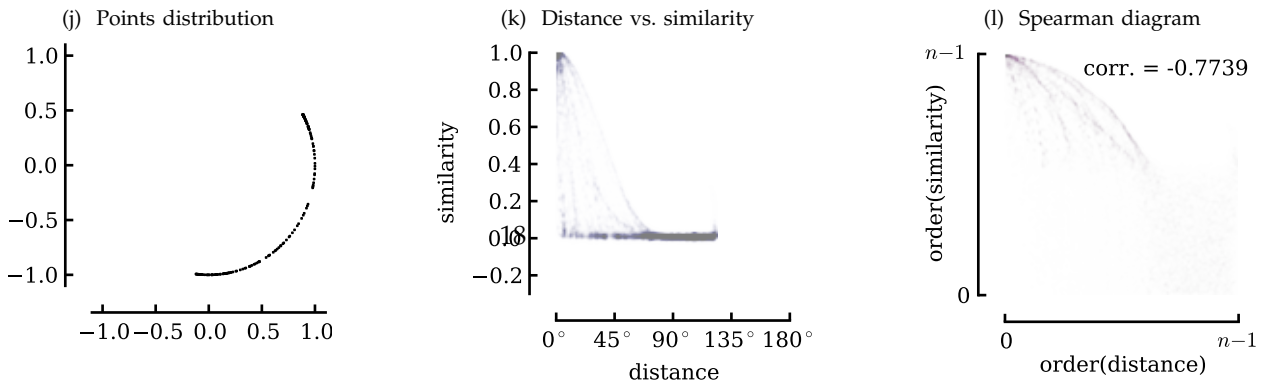
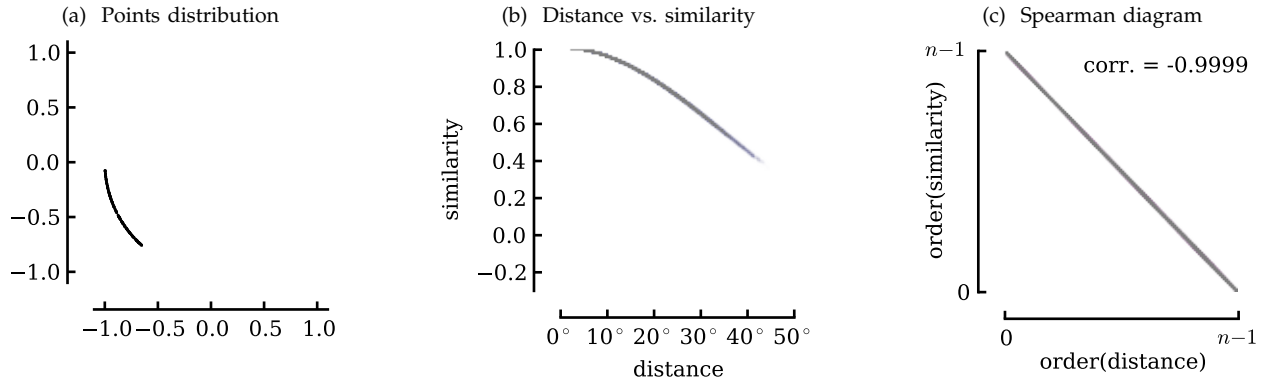
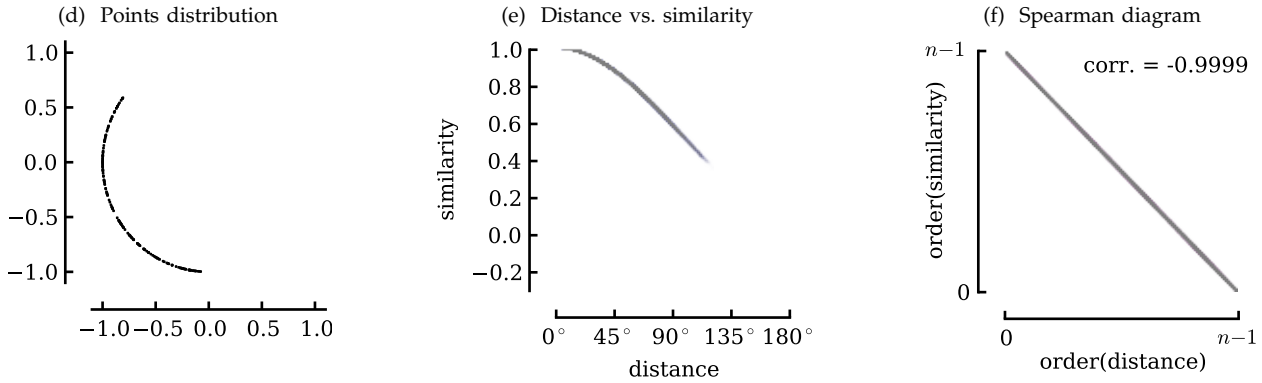


Fig. B-13: Random distribution on $\mathbb{S}^2$, 45deg fov, f_{smooth}

Ground truth



SKv



$\text{MDS}_{\mathbb{S}^n}$

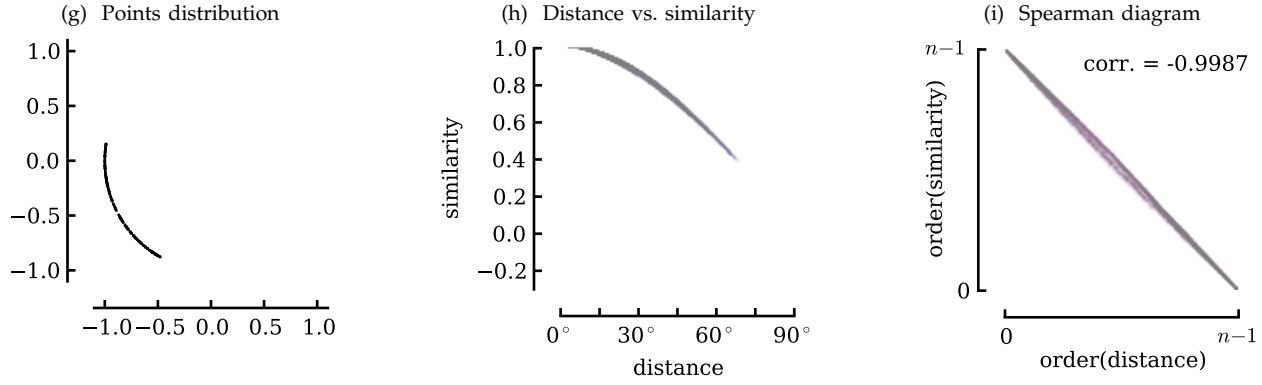
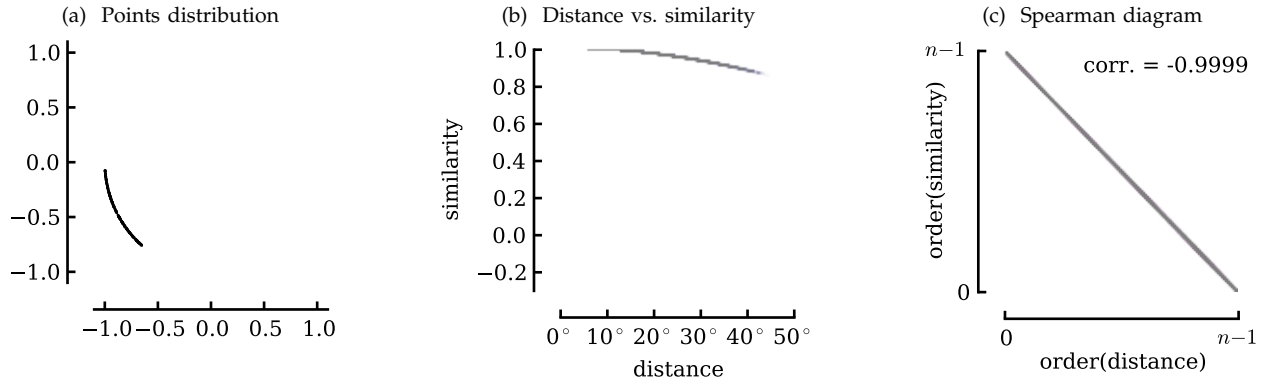
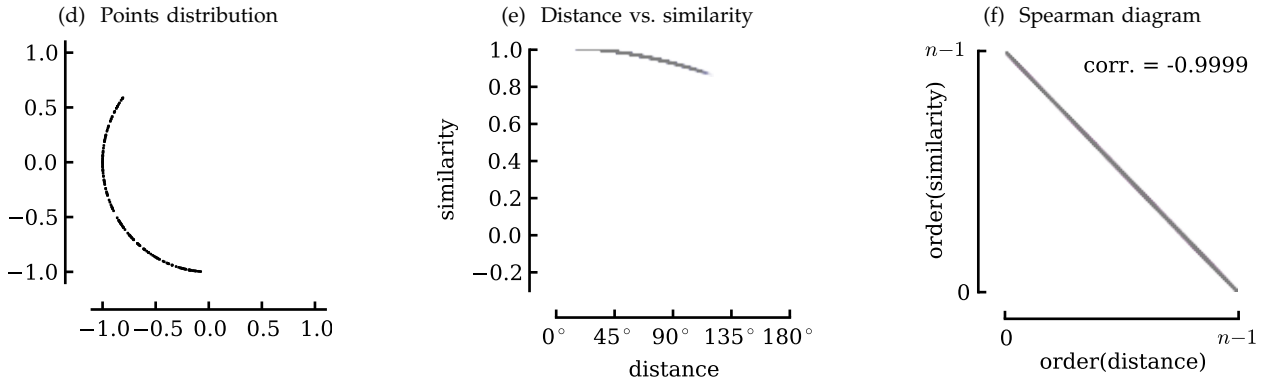


Fig. B-14: Random distribution on $\mathbb{S}^2$, 45deg fov, f_{lin}

Ground truth



SKv



$\text{MDS}_{\mathbb{S}^n}$

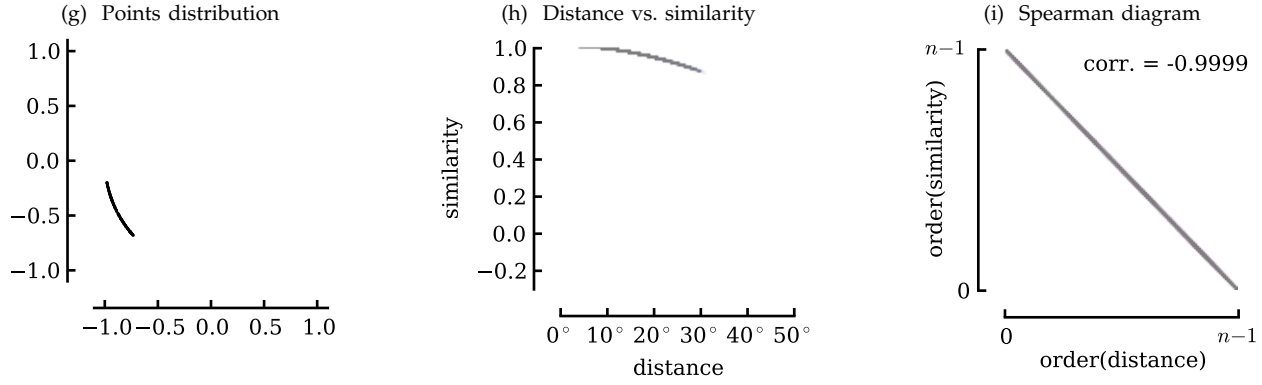
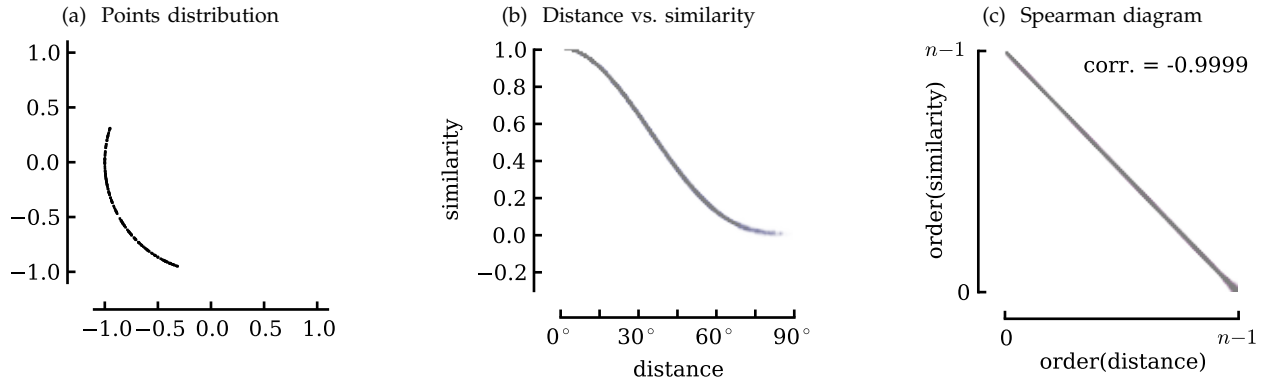
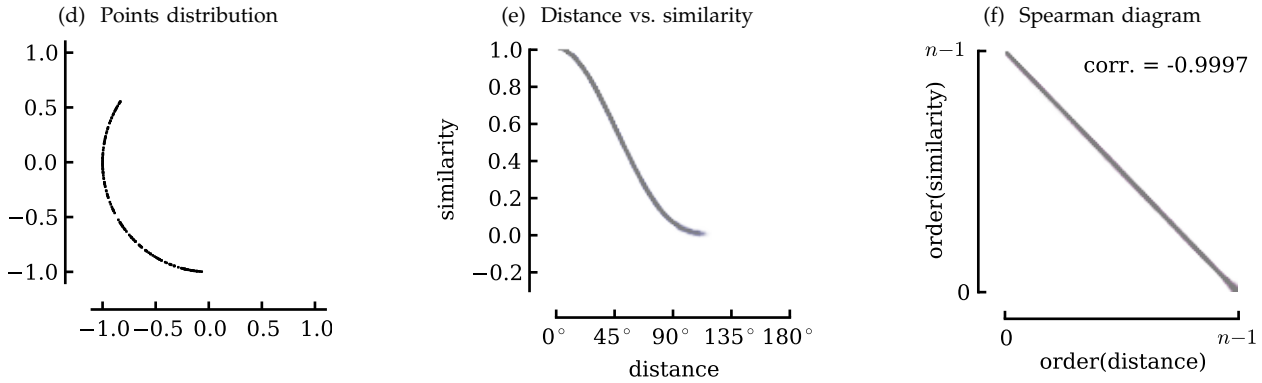


Fig. B-15: Random distribution on $\mathbb{S}^2$, 45deg fov, f_{smooth}

Ground truth



SKv



$\text{MDS}_{\mathbb{S}^n}$

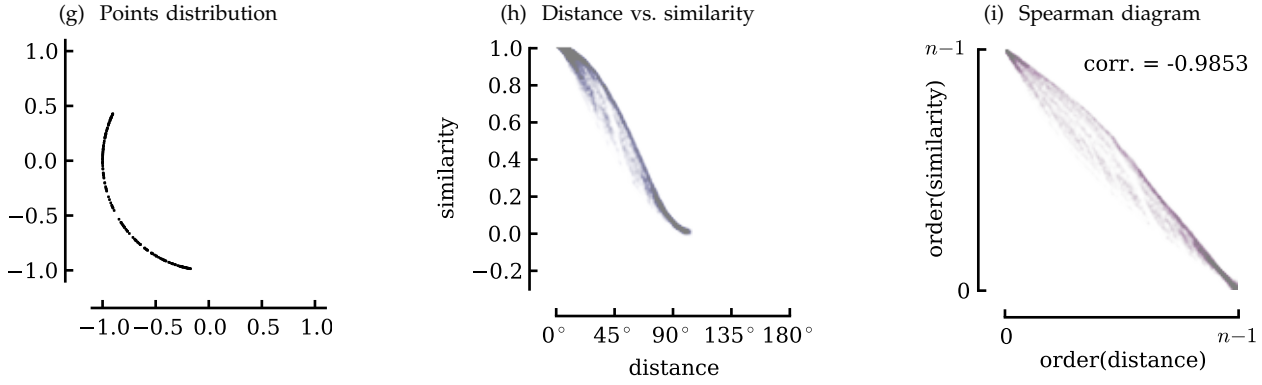
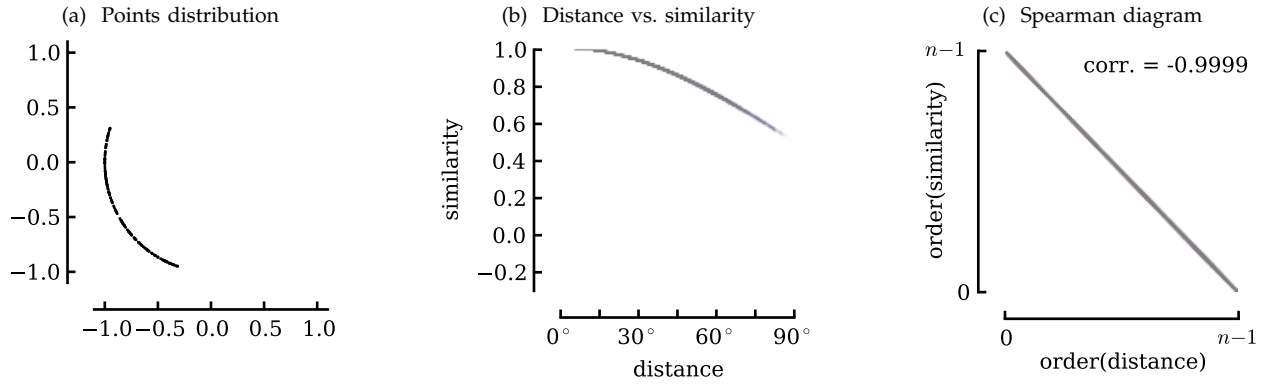
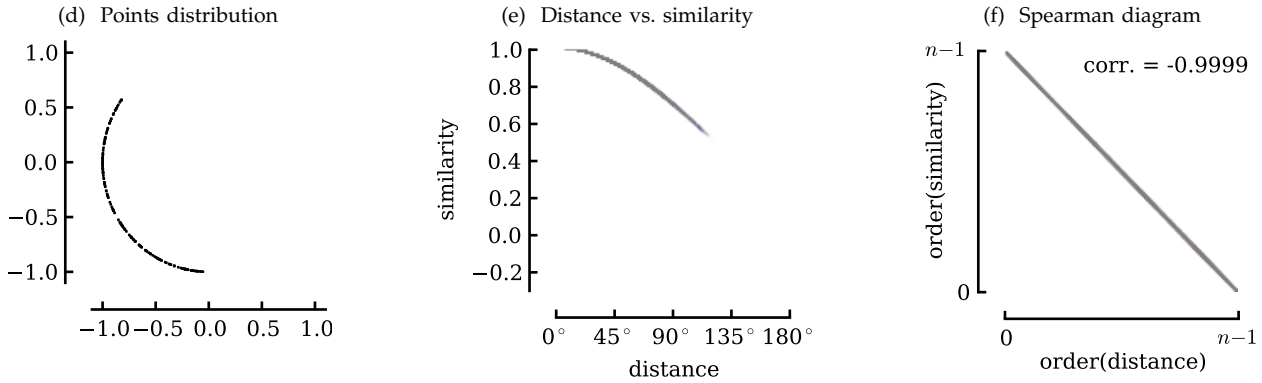


Fig. B-16: Random distribution on $\mathbb{S}^2$, 45deg fov, f_{lin}

Ground truth



SKv



$\text{MDS}_{\mathbb{S}^n}$

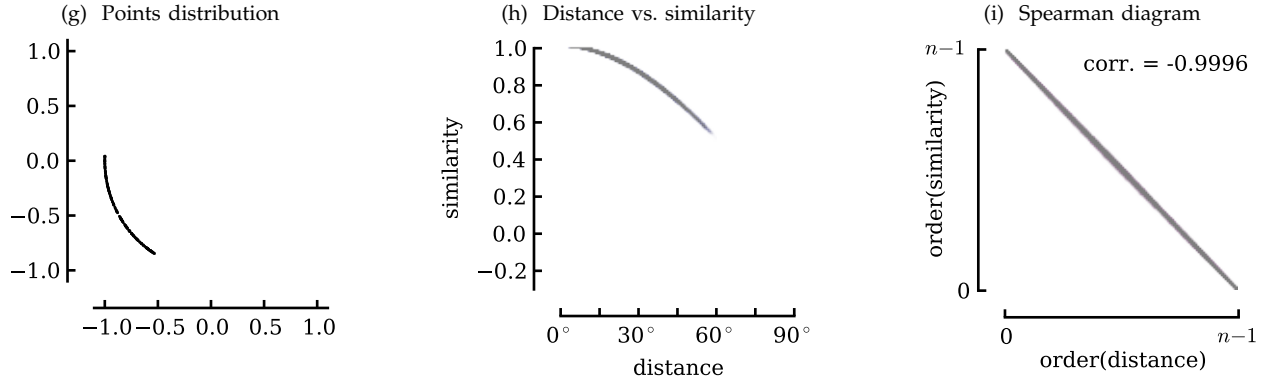
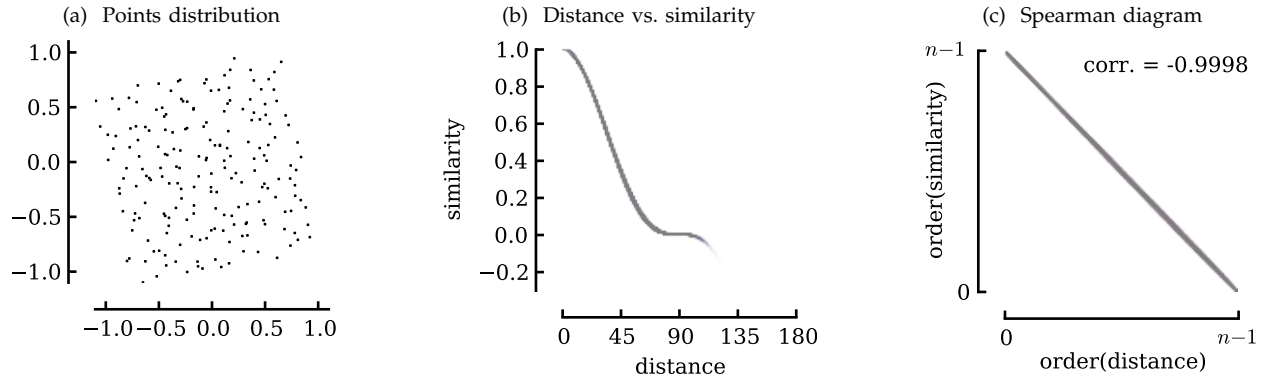
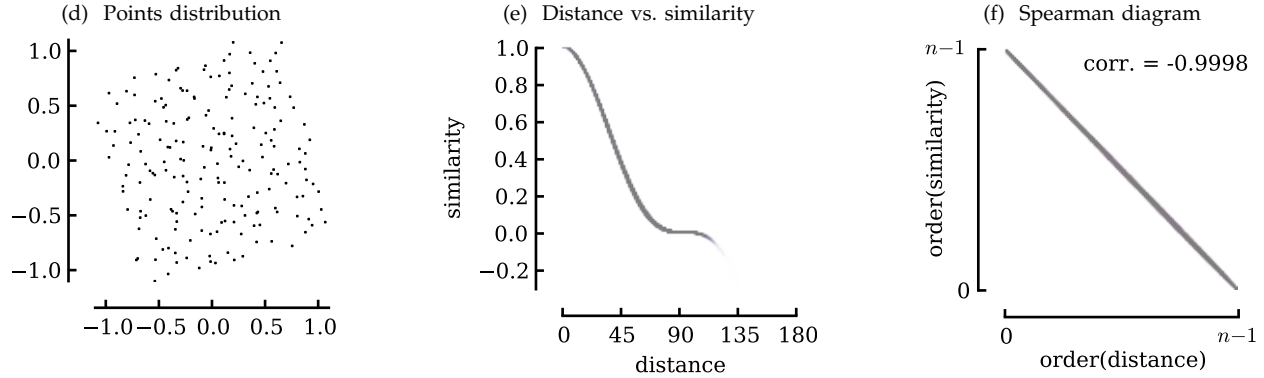


Fig. B-17: Random distribution on a square, f_{smooth}

Ground truth



SKv



$\mathsf{MDS}_{\mathbb{R}^n}$

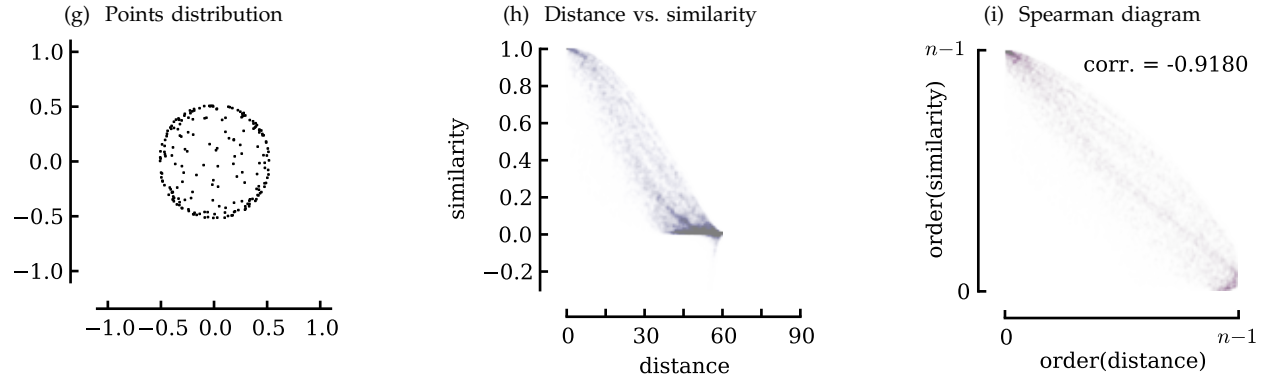
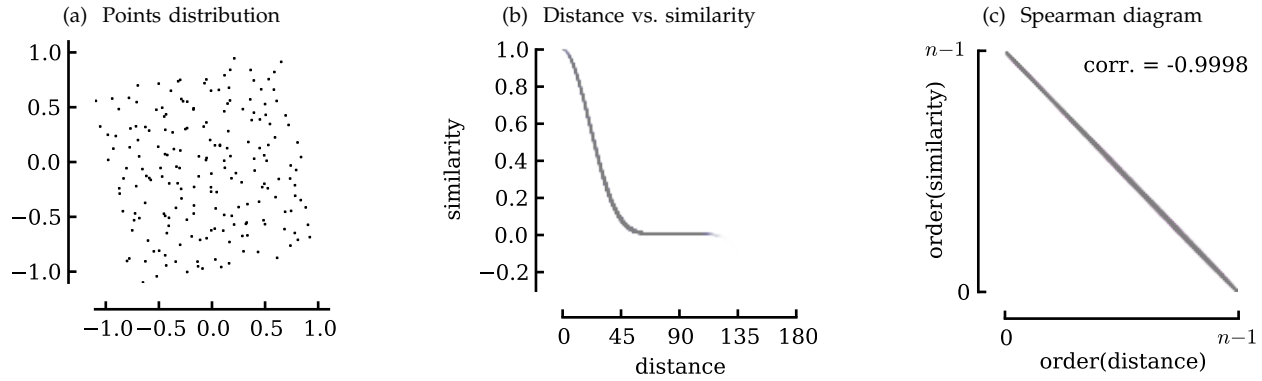
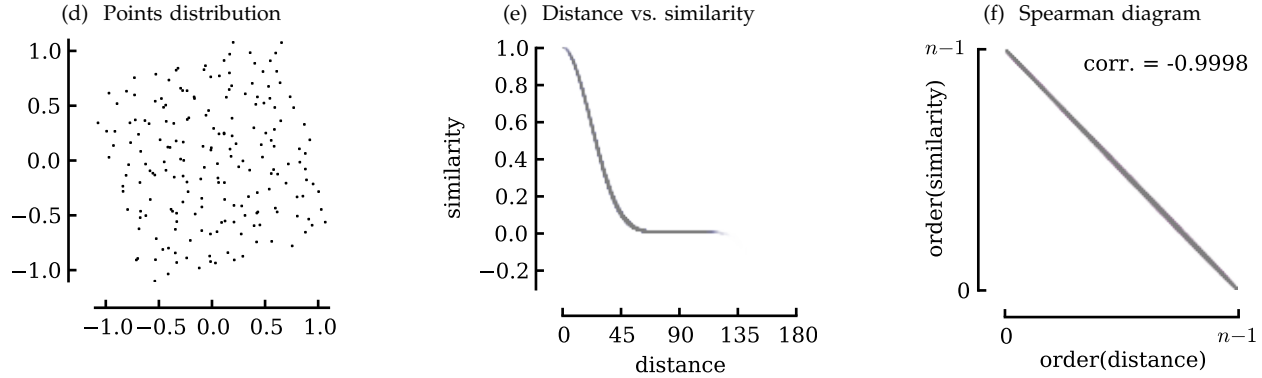


Fig. B-18: Random distribution on a square, f_{steep}

Ground truth



SKv



$\text{MDS}_{\mathbb{R}^n}$

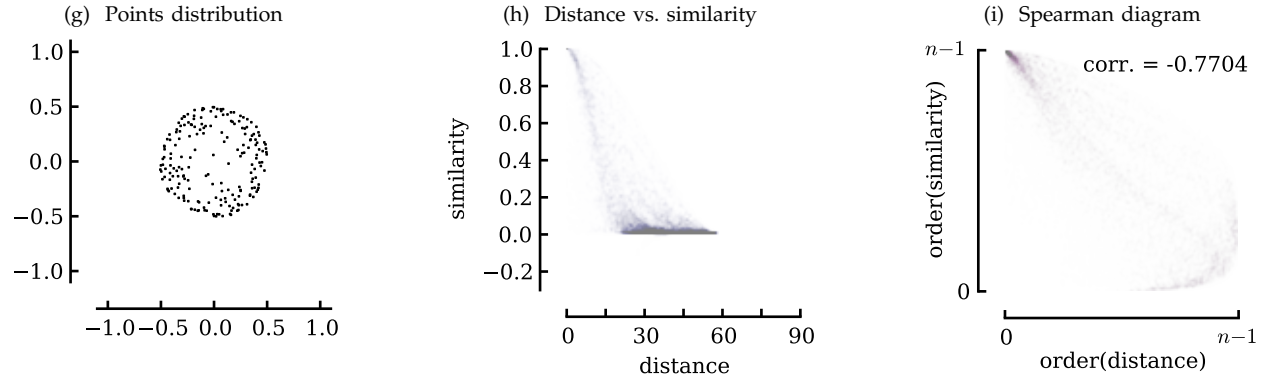
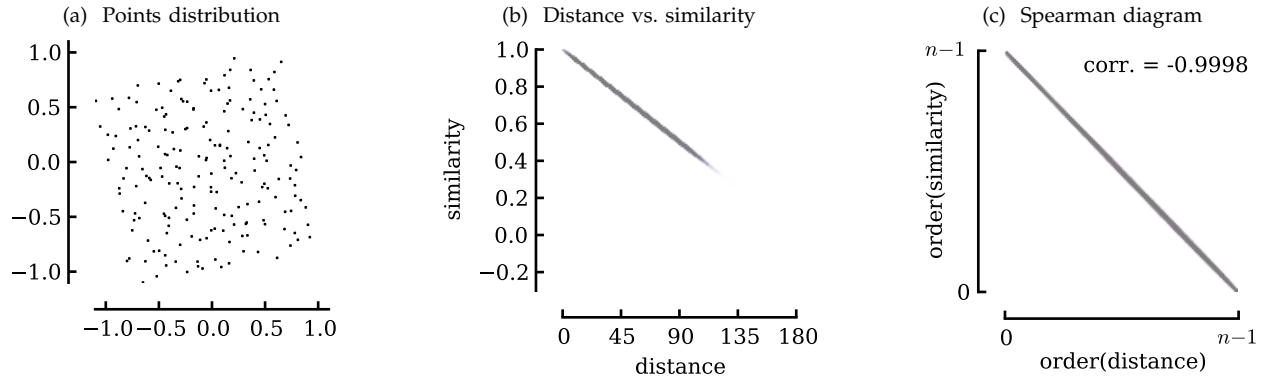
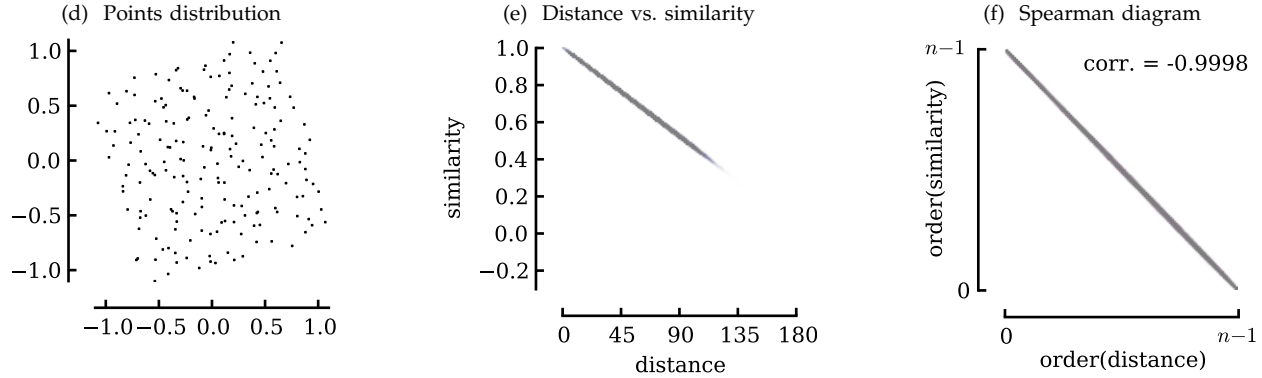


Fig. B-19: Random distribution on a square, f_{lin}

Ground truth



SKv



$\mathsf{MDS}_{\mathbb{R}^n}$