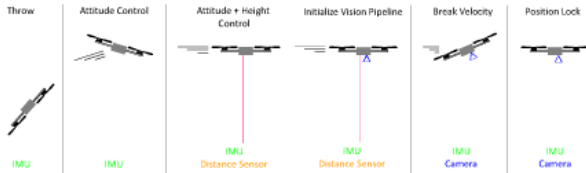


Automatic Re-Initialization and Failure Recovery for Aggressive Flight with a Monocular Vision-Based Quadrotor

Recovery Stages



State Estimation

IMU-Based Attitude Estimation

$$\begin{aligned}
 \dot{\mathbf{q}} &= \mathbf{q} \otimes \boldsymbol{\omega} \\
 \mathbf{q} &= \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} \in \mathbb{S}^3 \\
 \boldsymbol{\omega} &= \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \in \mathbb{R}^3 \\
 \mathbf{q} &= \begin{bmatrix} \cos(\frac{\theta}{2}) \cos(\frac{\phi}{2}) \cos(\frac{\psi}{2}) \\ \cos(\frac{\theta}{2}) \cos(\frac{\phi}{2}) \sin(\frac{\psi}{2}) \\ \cos(\frac{\theta}{2}) \sin(\frac{\phi}{2}) \cos(\frac{\psi}{2}) \\ \cos(\frac{\theta}{2}) \sin(\frac{\phi}{2}) \sin(\frac{\psi}{2}) \\ \sin(\frac{\theta}{2}) \cos(\frac{\phi}{2}) \cos(\frac{\psi}{2}) \\ \sin(\frac{\theta}{2}) \cos(\frac{\phi}{2}) \sin(\frac{\psi}{2}) \\ \sin(\frac{\theta}{2}) \sin(\frac{\phi}{2}) \cos(\frac{\psi}{2}) \\ \sin(\frac{\theta}{2}) \sin(\frac{\phi}{2}) \sin(\frac{\psi}{2}) \end{bmatrix}
 \end{aligned}$$

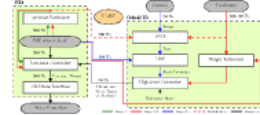
Vision-Based State Estimation

Control

Dynamical Model

$$\begin{aligned}
 \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\
 \mathbf{A} &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\
 \mathbf{B} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 \mathbf{x} &= \begin{bmatrix} x \\ y \\ z \\ \theta \end{bmatrix} \\
 \mathbf{u} &= \begin{bmatrix} u_x \\ u_y \\ u_z \\ \tau \end{bmatrix}
 \end{aligned}$$

Controller Overview



Position Controller

$$\begin{aligned}
 \dot{\mathbf{x}}_d &= \mathbf{K}_p(\mathbf{x}_d - \mathbf{x}) + \dot{\mathbf{x}}_d \\
 \mathbf{u} &= \mathbf{K}_v(\dot{\mathbf{x}}_d - \dot{\mathbf{x}}) + \dot{\mathbf{x}}_d
 \end{aligned}$$

Roll and Pitch Controller

$$\begin{aligned}
 \ddot{\theta} &= -\mathbf{K}_p(\theta - \theta_d) - \dot{\theta} + \ddot{\theta}_d \\
 \ddot{\phi} &= -\mathbf{K}_p(\phi - \phi_d) - \dot{\phi} + \ddot{\phi}_d
 \end{aligned}$$

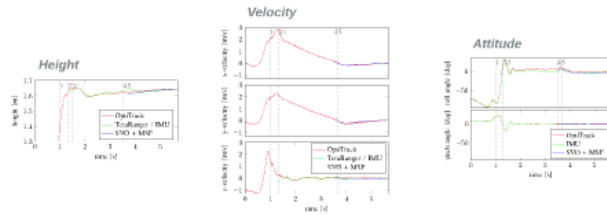
Yaw Controller

$$\begin{aligned}
 \ddot{\psi} &= -\mathbf{K}_p(\psi - \psi_d) - \dot{\psi} + \ddot{\psi}_d \\
 \tau &= \mathbf{J}^{-1}(\mathbf{u}_r, \mathbf{u}_p, \dot{\psi}_d)
 \end{aligned}$$

Low-Level Controller

$$\begin{aligned}
 \ddot{u}_x &= -\mathbf{K}_p(u_x - u_{x,d}) - \dot{u}_x + \ddot{u}_{x,d} \\
 \ddot{u}_y &= -\mathbf{K}_p(u_y - u_{y,d}) - \dot{u}_y + \ddot{u}_{y,d} \\
 \ddot{u}_z &= -\mathbf{K}_p(u_z - u_{z,d}) - \dot{u}_z + \ddot{u}_{z,d}
 \end{aligned}$$

Results



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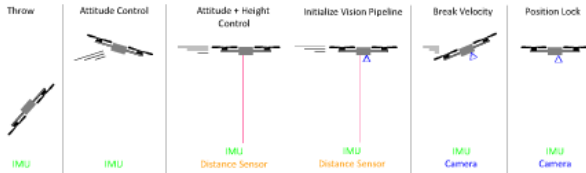
<http://rpg.ifi.uzh.ch>

References

- [1] C. Forster, M. Pizzoli, and D. Scaramuzza. *SWH: Fast semi-direct monocular visual odometry*. In *IEEE Int. Conf. on Robotics and Automation (ICRA)*, 2014.
- [2] S. Lynen, M. Achtelik, S. Weiss, M. Cam, and R. Siegwart. *A robust and modular multi-sensor fusion approach applied to MAV navigation*. in *IEEE/RSJ Int. Conf. on Intelligent Robots and Systems (IROS)*, 2013.
- [3] D. Droukas, M. Hahn, and R. D'Heckia. *Nonlinear quadcopter attitude control*. Department of Mechanical and Process Engineering, ETHZ, Tech. Rep., Oct. 2013.
- [4] C. Mayhew, R. Sanfelix, and A. Torn. *Quaternion-based hybrid control for robust global attitude tracking*. *Automatic Control, IEEE Transactions on*, vol. 56, no. 11, pp. 2558–2569, Nov 2011.
- [5] N. Trautman and S. I. Roumeliotis. *Indirect Kalman filter for 3D attitude estimation*. University of Minnesota, Dept. of Comp. Sci. & Eng. Tech. Rep. 2005.003, Mar 2005.
- [6] M. Faessler, F. Fontana, C. Forster, E. Moggio, M. Pizzoli, and D. Scaramuzza. *Autonomous, vision-based flight and live dense 3D mapping with a quadrotor MAV*. *J. of Field Robotics*, 2015.

Automatic Re-Initialization and Failure Recovery for Aggressive Flight with a Monocular Vision-Based Quadrotor

Recovery Stages



State Estimation

IMU-Based Attitude Estimation

$$\begin{aligned}
 \dot{q} &= \frac{1}{2} \begin{bmatrix} 2q_1 & 2q_2 & 2q_3 & 2q_4 \\ -q_2 q_3 - q_1 q_4 & q_1 q_3 - q_2 q_4 & q_1 q_4 + q_2 q_3 & q_2 q_4 - q_1 q_3 \end{bmatrix} \omega \\
 \dot{q} &= \frac{1}{2} \begin{bmatrix} 2q_1 & 2q_2 & 2q_3 & 2q_4 \\ -q_2 q_3 - q_1 q_4 & q_1 q_3 - q_2 q_4 & q_1 q_4 + q_2 q_3 & q_2 q_4 - q_1 q_3 \end{bmatrix} \omega \\
 \dot{q} &= \frac{1}{2} \begin{bmatrix} 2q_1 & 2q_2 & 2q_3 & 2q_4 \\ -q_2 q_3 - q_1 q_4 & q_1 q_3 - q_2 q_4 & q_1 q_4 + q_2 q_3 & q_2 q_4 - q_1 q_3 \end{bmatrix} \omega
 \end{aligned}$$

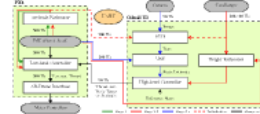
Vision-Based State Estimation

Control

Dynamical Model

$$\begin{aligned}
 \dot{x} &= v \\
 \dot{v} &= \frac{1}{m} \begin{bmatrix} -m v_x \\ -m v_y \\ -m v_z \\ -m \omega_x \\ -m \omega_y \\ -m \omega_z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\
 \dot{\theta} &= \omega
 \end{aligned}$$

Controller Overview



Position Controller

$$\begin{aligned}
 \ddot{x}_d &= \ddot{x}_d \\
 \ddot{y}_d &= \ddot{y}_d \\
 \ddot{z}_d &= \ddot{z}_d
 \end{aligned}$$

Roll and Pitch Controller

$$\begin{aligned}
 \ddot{\phi}_d &= \ddot{\phi}_d \\
 \ddot{\theta}_d &= \ddot{\theta}_d
 \end{aligned}$$

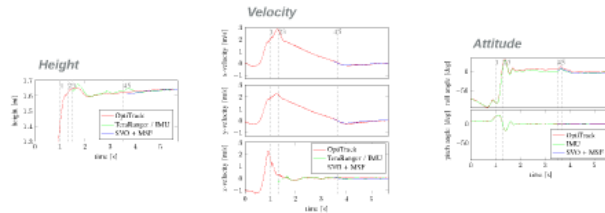
Yaw Controller

$$\begin{aligned}
 \ddot{\psi}_d &= \ddot{\psi}_d
 \end{aligned}$$

Low-Level Controller

$$\begin{aligned}
 \ddot{u}_x &= \ddot{u}_x \\
 \ddot{u}_y &= \ddot{u}_y \\
 \ddot{u}_z &= \ddot{u}_z
 \end{aligned}$$

Results



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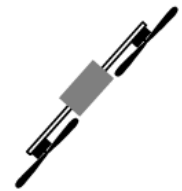
<http://rpg.ifi.uzh.ch>

References

- [1] C. Forster, M. Pizzoli, and D. Scaramuzza, *SWH: Fast semi-direct monocular visual odometry*, in *IEEE Int. Conf. on Robotics and Automation (ICRA)*, 2014.
- [2] S. Lynen, M. Achtelik, S. Weiss, M. Cam, and R. Siegwart, *A robust and modular multi-sensor fusion approach applied to MAV navigation*, in *IEEE/RSJ Int. Conf. on Intelligent Robots and Systems (IROS)*, 2013.
- [3] D. Droukas, M. Hahn, and R. D'Heck, *Nonlinear quadcopter attitude control*, Department of Mechanical and Process Engineering, ETHZ, Tech. Rep., Oct. 2013.
- [4] C. Mayhew, R. Sanfelix, and A. Torn, *Quaternion-based hybrid control for robust global attitude tracking*, *Automatic Control, IEEE Transactions on*, vol. 56, no. 11, pp. 2558–2569, Nov 2011.
- [5] N. Trautman and S. I. Roumeliotis, *Indirect Kalman filter for 3D attitude estimation*, University of Minnesota, Dept. of Comp. Sci. & Eng. Tech. Rep., 2005, 003, Mar 2005.
- [6] M. Faessler, F. Fontana, C. Forster, E. Moggio, M. Pizzoli, and D. Scaramuzza, *Autonomous, vision-based flight and live dense 3D mapping with a quadrotor MAV*, *J. of Field Robotics*, 2015.

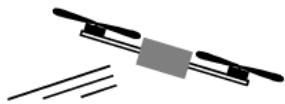
Recovery Stages

Throw



IMU

Attitude Control



IMU

Attitude + Height Control



IMU
Distance Sensor

Initialize Vision Pipeline



IMU
Distance Sensor

Break Velocity



IMU
Camera

Position Lock



IMU
Camera

State Estimation

State Estimation

IMU-Based Attitude Estimation

$$\hat{\mathbf{q}}_{pred}(k) = \left(\mathbf{I}_4 \cdot \cos\left(\frac{\|\hat{\boldsymbol{\omega}}\| \Delta t}{2}\right) + \frac{2}{\|\hat{\boldsymbol{\omega}}\|} \cdot \boldsymbol{\Lambda}(\hat{\boldsymbol{\omega}}) \cdot \sin\left(\frac{\|\hat{\boldsymbol{\omega}}\| \Delta t}{2}\right) \right) \cdot \hat{\mathbf{q}}(k-1) \quad \|\hat{\boldsymbol{\omega}}\| \neq 0$$

$$\beta = \arccos\left(\frac{{}_B \hat{\mathbf{e}}_{z,pred}^B \cdot \hat{\mathbf{a}}}{\|\hat{\mathbf{a}}\|}\right)$$

$${}_B \mathbf{h} = \frac{{}_B \hat{\mathbf{a}} \times_B \hat{\mathbf{e}}_{z,pred}^B}{\|{}_B \hat{\mathbf{a}} \times_B \hat{\mathbf{e}}_{z,pred}^B\|}$$

$$\mathbf{q}_{corr} = \begin{bmatrix} \cos(k_{corr} \cdot \frac{\beta}{2}) \\ {}_B \mathbf{h} \sin(k_{corr} \cdot \frac{\beta}{2}) \end{bmatrix}$$

$$\hat{\mathbf{q}}(k) = \hat{\mathbf{q}}_{pred}(k) \otimes \mathbf{q}_{corr}$$

$$\|\hat{\boldsymbol{\omega}}\| < 0.5 \text{ rad s}^{-1}$$

$$\|\hat{\mathbf{a}}\| - g < 1.0 \text{ m s}^{-2}$$

Vision-Based State Estimation

IMU-Based Attitude Estimation

$$\hat{\mathbf{q}}_{pred}(k) = \left(\mathbf{I}_4 \cdot \cos\left(\frac{\|\hat{\boldsymbol{\omega}}\| \Delta t}{2}\right) + \frac{2}{\|\hat{\boldsymbol{\omega}}\|} \cdot \boldsymbol{\Lambda}(\hat{\boldsymbol{\omega}}) \cdot \sin\left(\frac{\|\hat{\boldsymbol{\omega}}\| \Delta t}{2}\right) \right) \cdot \hat{\mathbf{q}}(k-1) \quad \|\hat{\boldsymbol{\omega}}\| \neq 0$$

$$\beta = \arccos\left({}_B \hat{\mathbf{e}}_{z,pred}^B \cdot \frac{\tilde{\mathbf{a}}}{\|\tilde{\mathbf{a}}\|} \right)$$

$${}_B \mathbf{h} = \frac{{}_B \tilde{\mathbf{a}} \times {}_B \hat{\mathbf{e}}_{z,pred}^B}{\|{}_B \tilde{\mathbf{a}} \times {}_B \hat{\mathbf{e}}_{z,pred}^B\|}$$

$$\mathbf{q}_{corr} = \begin{bmatrix} \cos(k_{corr} \cdot \frac{\beta}{2}) \\ {}_B \mathbf{h} \sin(k_{corr} \cdot \frac{\beta}{2}) \end{bmatrix}$$

$$\hat{\mathbf{q}}(k) = \hat{\mathbf{q}}_{pred}(k) \otimes \mathbf{q}_{corr}$$

$$\|\tilde{\boldsymbol{\omega}}\| < 0.5 \text{ rad s}^{-1}$$

$$\| \|\tilde{\mathbf{a}}\| - g \| < 1.0 \text{ m s}^{-2}$$

Vision-Based State Estimation

Control

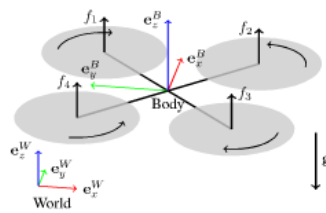
Dynamical Model

$$\begin{aligned} \dot{\mathbf{r}} &= \mathbf{v} \\ \dot{\mathbf{v}} &= \mathbf{g} + \mathbf{q} \odot \mathbf{c} \\ \dot{\mathbf{q}} &= \mathbf{\Lambda}(\boldsymbol{\omega}) \cdot \mathbf{q} \\ \dot{\boldsymbol{\omega}} &= \mathbf{J}^{-1} \cdot (\boldsymbol{\tau} - \boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega}) \end{aligned}$$

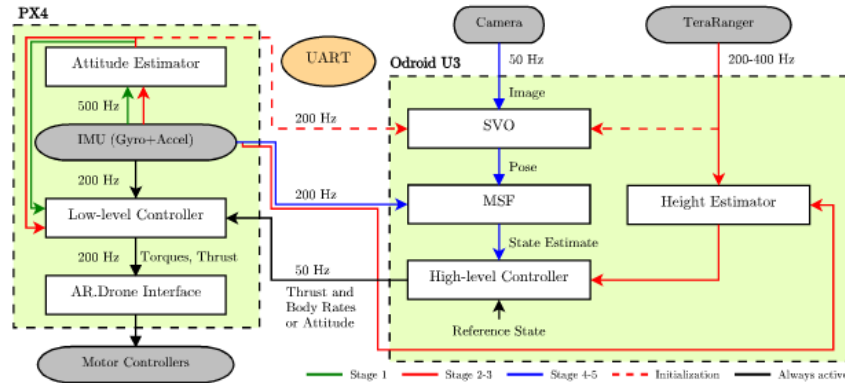
$$\mathbf{\Lambda}(\boldsymbol{\omega}) = \frac{1}{2} \begin{bmatrix} 0 & -p & -q & -r \\ p & 0 & r & -q \\ q & -r & 0 & p \\ r & q & -p & 0 \end{bmatrix}$$

$$mc = f_1 + f_2 + f_3 + f_4$$

$$\boldsymbol{\tau} = \begin{bmatrix} \frac{\sqrt{2}}{2}l(f_1 - f_2 - f_3 + f_4) \\ \frac{\sqrt{2}}{2}l(-f_1 - f_2 + f_3 + f_4) \\ \kappa(f_1 - f_2 + f_3 - f_4) \end{bmatrix}$$



Controller Overview



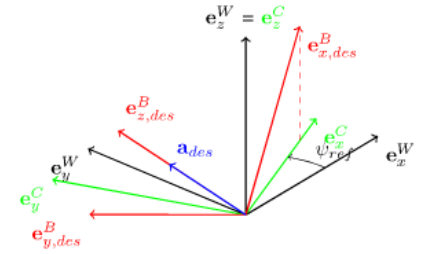
Position Controller

$$\mathbf{a}_{des} = \mathbf{P}_{pos} \cdot (\mathbf{r}_{ref} - \hat{\mathbf{r}}) + \mathbf{D}_{pos} \cdot (\mathbf{v}_{ref} - \hat{\mathbf{v}}) + \mathbf{a}_{ref} - \mathbf{g}$$

$$\mathbf{c}_{des} = \mathbf{a}_{des} \cdot \mathbf{e}_z^B$$

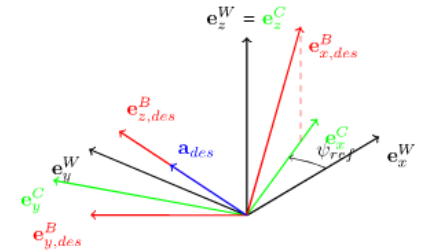
Roll and Pitch Controller

$$\begin{aligned} \hat{\mathbf{e}}_z^B &= \hat{\mathbf{q}} \otimes [0 \ 0 \ 1]^T \\ \mathbf{e}_{z,des}^B &= \frac{\mathbf{a}_{des}}{\|\mathbf{a}_{des}\|} \\ \alpha &= \arccos(\hat{\mathbf{e}}_z^B \cdot \mathbf{e}_{z,des}^B) \\ \mathbf{n} &= \frac{\hat{\mathbf{e}}_z^B \times \mathbf{e}_{z,des}^B}{\|\hat{\mathbf{e}}_z^B \times \mathbf{e}_{z,des}^B\|} \\ \mathbf{B}\mathbf{n} &= \hat{\mathbf{q}}^{-1} \odot \mathbf{n} \\ \mathbf{q}_{e,rp} &= \begin{bmatrix} \cos(\frac{\alpha}{2}) \\ \mathbf{B}\mathbf{n} \sin(\frac{\alpha}{2}) \end{bmatrix} \\ \begin{bmatrix} p \\ q \end{bmatrix}_{des} &= \begin{cases} 2 \cdot p_{rp} \cdot \mathbf{q}_{e,rp}^{(x,y)} & \text{if } \mathbf{q}_{e,rp}^{(w)} \geq 0 \\ -2 \cdot p_{rp} \cdot \mathbf{q}_{e,rp}^{(x,y)} & \text{if } \mathbf{q}_{e,rp}^{(w)} < 0 \end{cases} \end{aligned}$$



Yaw Controller

$$\begin{aligned} \mathbf{e}_x^C &= [\cos(\psi_{ref}) \ \sin(\psi_{ref}) \ 0]^T \\ \mathbf{e}_y^C &= [-\sin(\psi_{ref}) \ \cos(\psi_{ref}) \ 0]^T \\ \mathbf{e}_{x,des}^B &= \frac{\mathbf{e}_y^C \times \mathbf{e}_{z,des}^B}{\|\mathbf{e}_y^C \times \mathbf{e}_{z,des}^B\|} \\ \mathbf{e}_{y,des}^B &= \frac{\mathbf{e}_{z,des}^B \times \mathbf{e}_{x,des}^B}{\|\mathbf{e}_{z,des}^B \times \mathbf{e}_{x,des}^B\|} \\ \mathbf{q}_{e,y} &= (\hat{\mathbf{q}} \otimes \mathbf{q}_{e,rp})^{-1} \otimes \mathbf{q}_{des} \\ r_{des} &= \begin{cases} 2 \cdot p_{yaw} \cdot \mathbf{q}_{e,y}^{(z)} & \text{if } \mathbf{q}_{e,y}^{(w)} \geq 0 \\ -2 \cdot p_{yaw} \cdot \mathbf{q}_{e,y}^{(z)} & \text{if } \mathbf{q}_{e,y}^{(w)} < 0 \end{cases} \end{aligned}$$



Low-Level Controller

$$\boldsymbol{\tau}_{des} = \mathbf{J} \cdot \mathbf{P}_{att} \cdot (\boldsymbol{\omega}_{des} - \hat{\boldsymbol{\omega}}) + \hat{\boldsymbol{\omega}} \times \mathbf{J}\hat{\boldsymbol{\omega}}$$

$$mc = f_1 + f_2 + f_3 + f_4$$

$$\boldsymbol{\tau} = \begin{bmatrix} \frac{\sqrt{2}}{2}l(f_1 - f_2 - f_3 + f_4) \\ \frac{\sqrt{2}}{2}l(-f_1 - f_2 + f_3 + f_4) \\ \kappa(f_1 - f_2 + f_3 - f_4) \end{bmatrix}$$

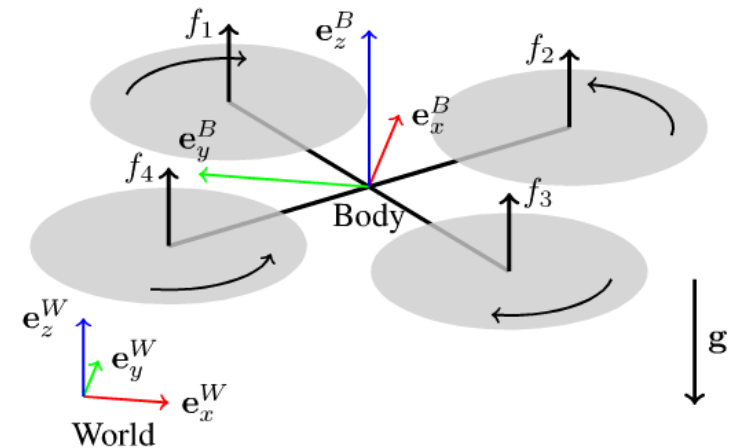
Dynamical Model

$$\begin{aligned} \dot{\mathbf{r}} &= \mathbf{v} \\ \dot{\mathbf{v}} &= \mathbf{g} + \mathbf{q} \odot \mathbf{c} \\ \dot{\mathbf{q}} &= \mathbf{\Lambda}(\boldsymbol{\omega}) \cdot \mathbf{q} \\ \dot{\boldsymbol{\omega}} &= \mathbf{J}^{-1} \cdot (\boldsymbol{\tau} - \boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega}) \end{aligned}$$

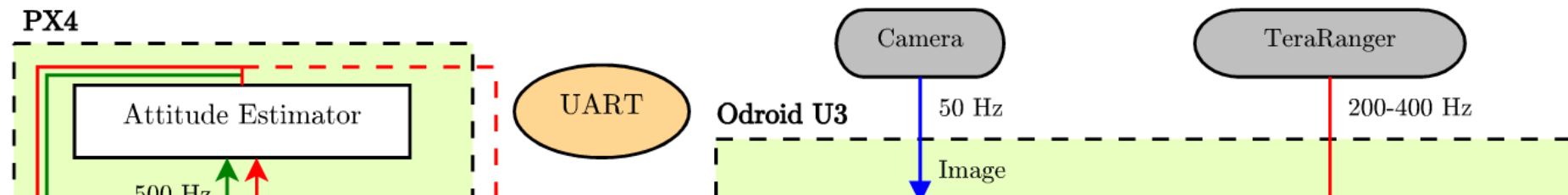
$$\mathbf{\Lambda}(\boldsymbol{\omega}) = \frac{1}{2} \begin{bmatrix} 0 & -p & -q & -r \\ p & 0 & r & -q \\ q & -r & 0 & p \\ r & q & -p & 0 \end{bmatrix}$$

$$mc = f_1 + f_2 + f_3 + f_4$$

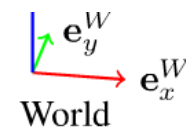
$$\boldsymbol{\tau} = \begin{bmatrix} \frac{\sqrt{2}}{2}l(f_1 - f_2 - f_3 + f_4) \\ \frac{\sqrt{2}}{2}l(-f_1 - f_2 + f_3 + f_4) \\ \kappa(f_1 - f_2 + f_3 - f_4) \end{bmatrix}$$



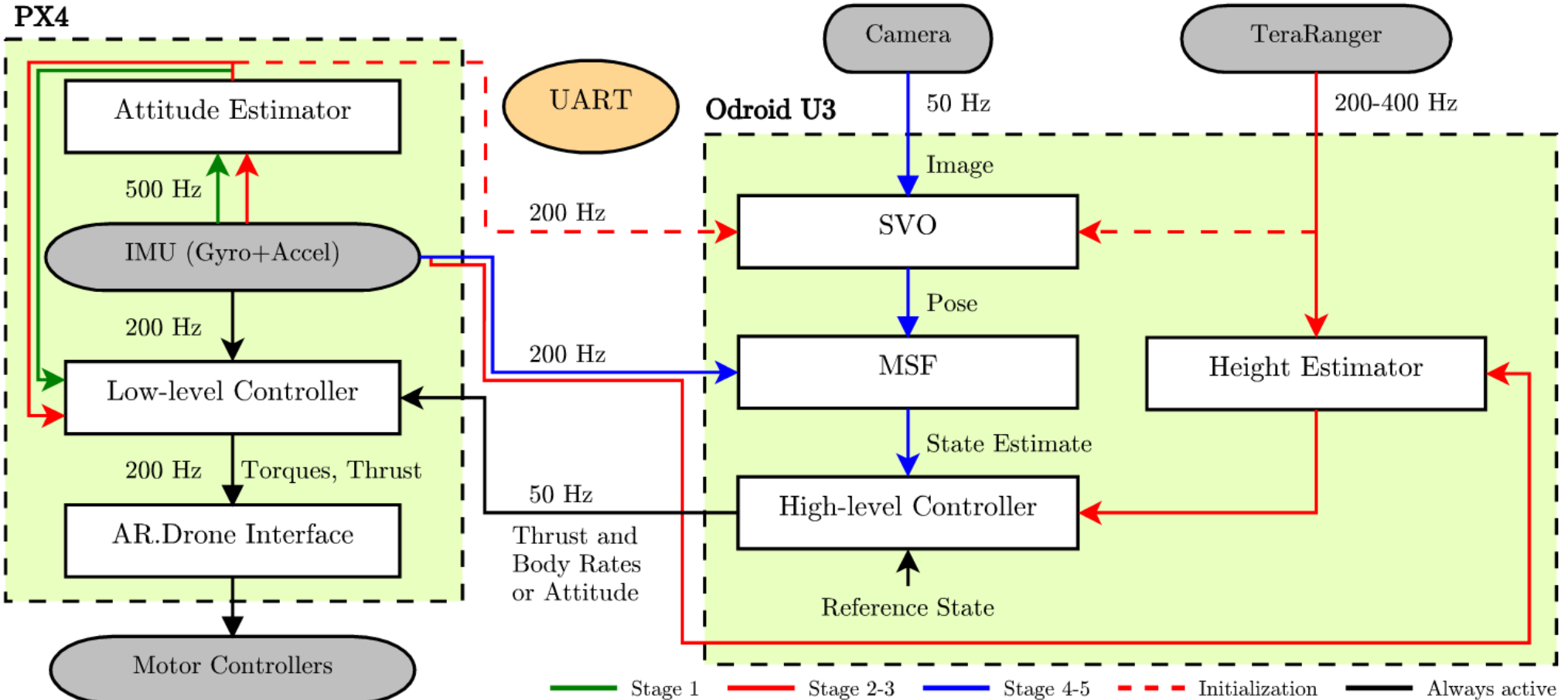
Controller Overview



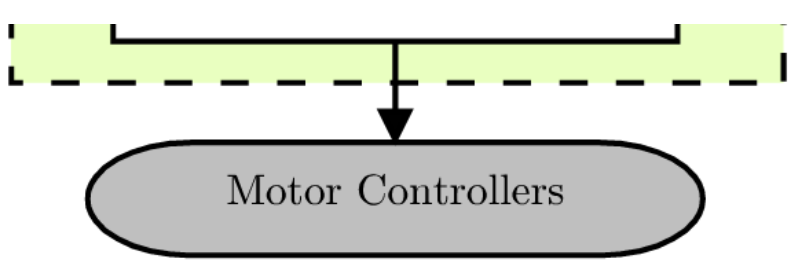
$$\tau = \begin{bmatrix} \frac{\sqrt{L}}{2} l (-f_1 - f_2 + f_3 + f_4) \\ \kappa (f_1 - f_2 + f_3 - f_4) \end{bmatrix}$$



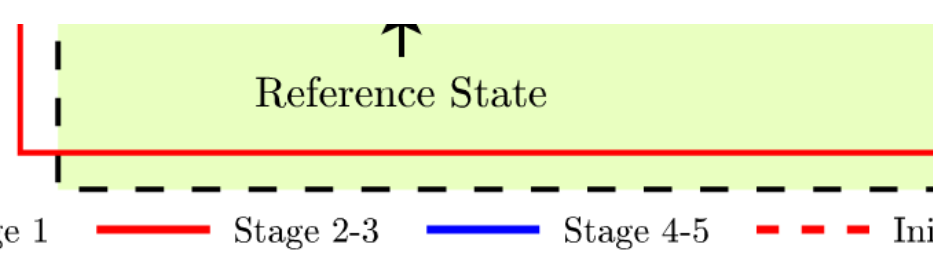
Controller Overview



Position Controller



Body States
or Attitude



Position Controller

$$\mathbf{a}_{des} = \mathbf{P}_{pos} \cdot (\mathbf{r}_{ref} - \hat{\mathbf{r}}) + \mathbf{D}_{pos} \cdot (\mathbf{v}_{ref} - \hat{\mathbf{v}}) + \mathbf{a}_{ref} - \mathbf{g}$$

$$c_{des} = \mathbf{a}_{des} \cdot \mathbf{e}_z^B$$

Roll and Pitch Controller

$$\hat{\mathbf{e}}_z^B = \hat{\mathbf{q}} \otimes [0 \ 0 \ 1]^T$$

$$\mathbf{e}_{z,des}^B = \frac{\mathbf{a}_{des}}{\|\mathbf{a}_{des}\|}$$

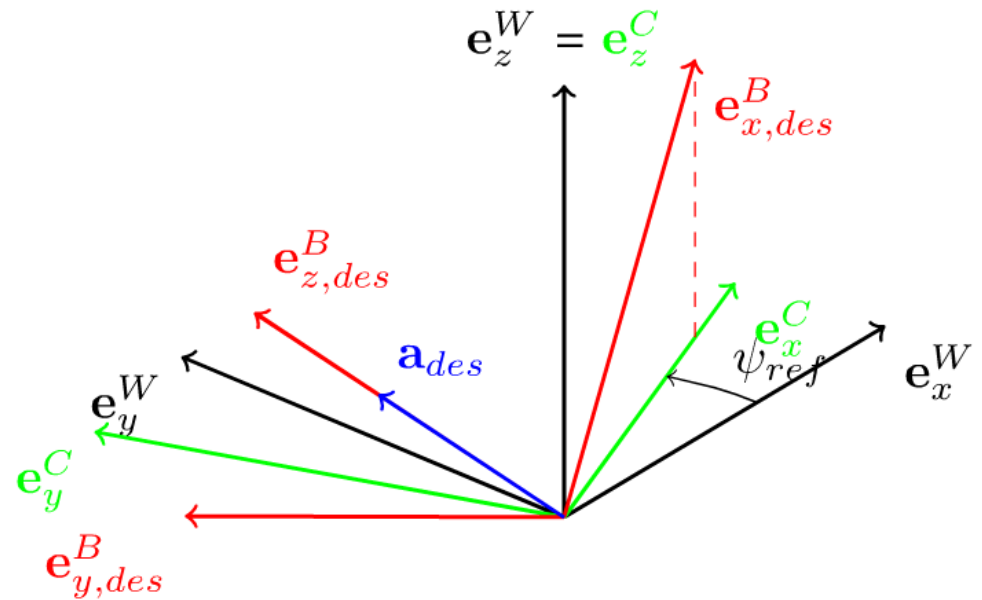
$$\alpha = \arccos(\hat{\mathbf{e}}_z^B \cdot \mathbf{e}_{z,des}^B)$$

$$\mathbf{n} = \frac{\hat{\mathbf{e}}_z^B \times \mathbf{e}_{z,des}^B}{\|\hat{\mathbf{e}}_z^B \times \mathbf{e}_{z,des}^B\|}$$

$${}_B\mathbf{n} = \hat{\mathbf{q}}^{-1} \odot \mathbf{n}$$

$$\mathbf{q}_{e,rp} = \begin{bmatrix} \cos(\frac{\alpha}{2}) \\ {}_B\mathbf{n} \sin(\frac{\alpha}{2}) \end{bmatrix}$$

$$\begin{bmatrix} p \\ q \end{bmatrix}_{des} = \begin{cases} 2 \cdot p_{rp} \cdot \mathbf{q}_{e,rp}^{(x,y)} & \text{if } \mathbf{q}_{e,rp}^{(w)} \geq 0 \\ -2 \cdot p_{rp} \cdot \mathbf{q}_{e,rp}^{(x,y)} & \text{if } \mathbf{q}_{e,rp}^{(w)} < 0 \end{cases}$$

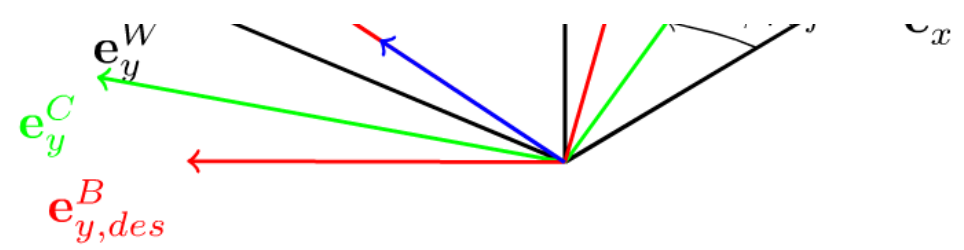


Yaw Controller

$$\mathbf{e}_x^C = [\cos(\psi_{ref}) \ \sin(\psi_{ref}) \ 0]^T$$

$$\mathbf{q}_{e,rp} = \begin{bmatrix} \cos(\frac{\alpha}{2}) \\ B \mathbf{n} \sin(\frac{\alpha}{2}) \end{bmatrix}$$

$$\begin{bmatrix} p \\ q \end{bmatrix}_{des} = \begin{cases} 2 \cdot p_{rp} \cdot \mathbf{q}_{e,rp}^{(x,y)} & \text{if } \mathbf{q}_{e,rp}^{(w)} \geq 0 \\ -2 \cdot p_{rp} \cdot \mathbf{q}_{e,rp}^{(x,y)} & \text{if } \mathbf{q}_{e,rp}^{(w)} < 0 \end{cases}$$



Yaw Controller

$$\mathbf{e}_x^C = [\cos(\psi_{ref}) \sin(\psi_{ref}) 0]^T$$

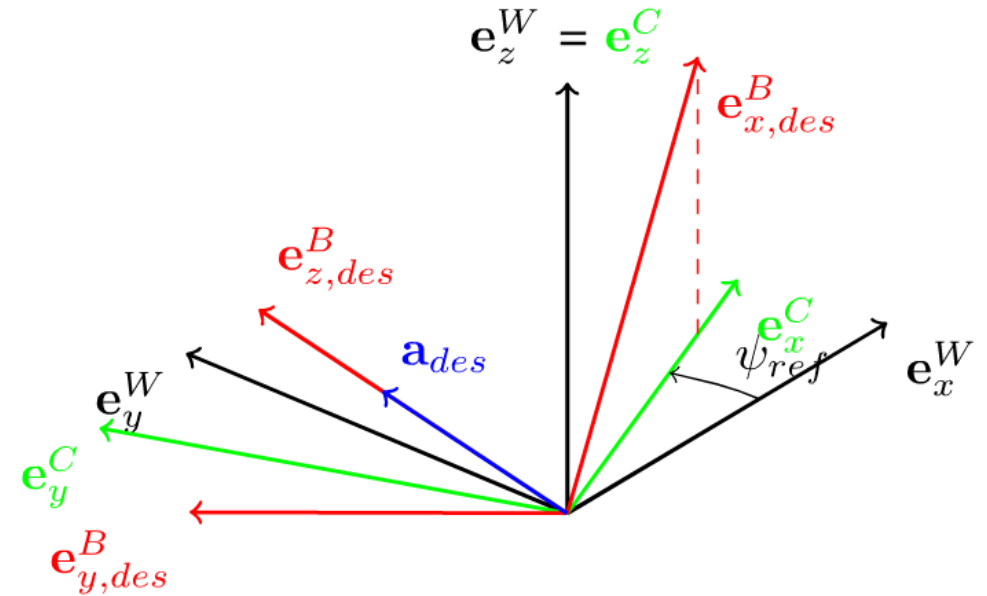
$$\mathbf{e}_y^C = [-\sin(\psi_{ref}) \cos(\psi_{ref}) 0]^T$$

$$\mathbf{e}_{x,des}^B = \frac{\mathbf{e}_y^C \times \mathbf{e}_{z,des}^B}{\|\mathbf{e}_y^C \times \mathbf{e}_{z,des}^B\|}$$

$$\mathbf{e}_{y,des}^B = \frac{\mathbf{e}_{z,des}^B \times \mathbf{e}_{x,des}^B}{\|\mathbf{e}_{z,des}^B \times \mathbf{e}_{x,des}^B\|}$$

$$\mathbf{q}_{e,y} = (\hat{\mathbf{q}} \otimes \mathbf{q}_{e,rp})^{-1} \otimes \mathbf{q}_{des}$$

$$r_{des} = \begin{cases} 2 \cdot p_{yaw} \cdot \mathbf{q}_{e,y}^{(z)} & \text{if } \mathbf{q}_{e,y}^{(w)} \geq 0 \\ -2 \cdot p_{yaw} \cdot \mathbf{q}_{e,y}^{(z)} & \text{if } \mathbf{q}_{e,y}^{(w)} < 0 \end{cases}$$



Low-Level Controller

Low-Level Controller

$$\boldsymbol{\tau}_{des} = \mathbf{J} \cdot \mathbf{P}_{att} \cdot (\boldsymbol{\omega}_{des} - \hat{\boldsymbol{\omega}}) + \hat{\boldsymbol{\omega}} \times \mathbf{J} \hat{\boldsymbol{\omega}}$$

$$mc = f_1 + f_2 + f_3 + f_4$$

$$\boldsymbol{\tau} = \begin{bmatrix} \frac{\sqrt{2}}{2} l (f_1 - f_2 - f_3 + f_4) \\ \frac{\sqrt{2}}{2} l (-f_1 - f_2 + f_3 + f_4) \\ \kappa (f_1 - f_2 + f_3 - f_4) \end{bmatrix}$$

$$\mathbf{a}_{des} = \mathbf{P}_{pos} \cdot (\mathbf{r}_{ref} - \hat{\mathbf{r}}) + \mathbf{D}_{pos} \cdot (\mathbf{v}_{ref} - \hat{\mathbf{v}}) + \mathbf{a}_{ref} - \mathbf{g}$$

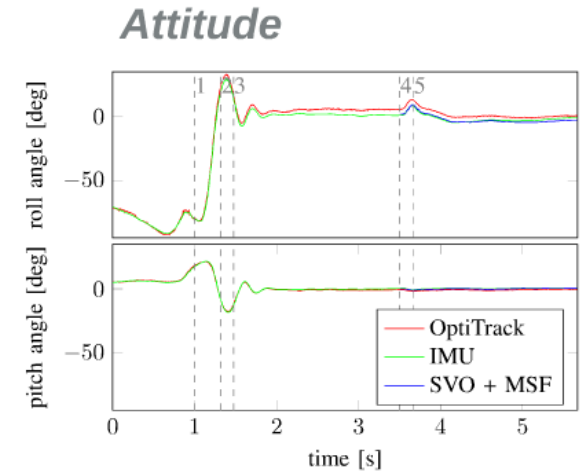
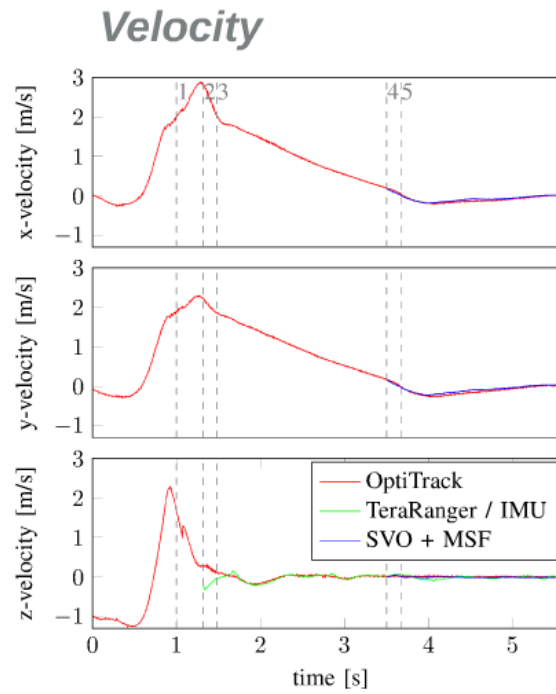
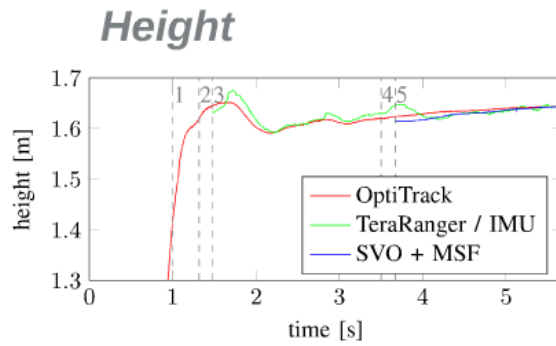
$$\mathbf{c}_{des} = \mathbf{a}_{des} \cdot \mathbf{e}_z^B$$

$$\boldsymbol{\tau}_{des} = \mathbf{J} \cdot \mathbf{P}_{att} \cdot (\boldsymbol{\omega}_{des} - \hat{\boldsymbol{\omega}}) + \hat{\boldsymbol{\omega}} \times \mathbf{J} \hat{\boldsymbol{\omega}}$$

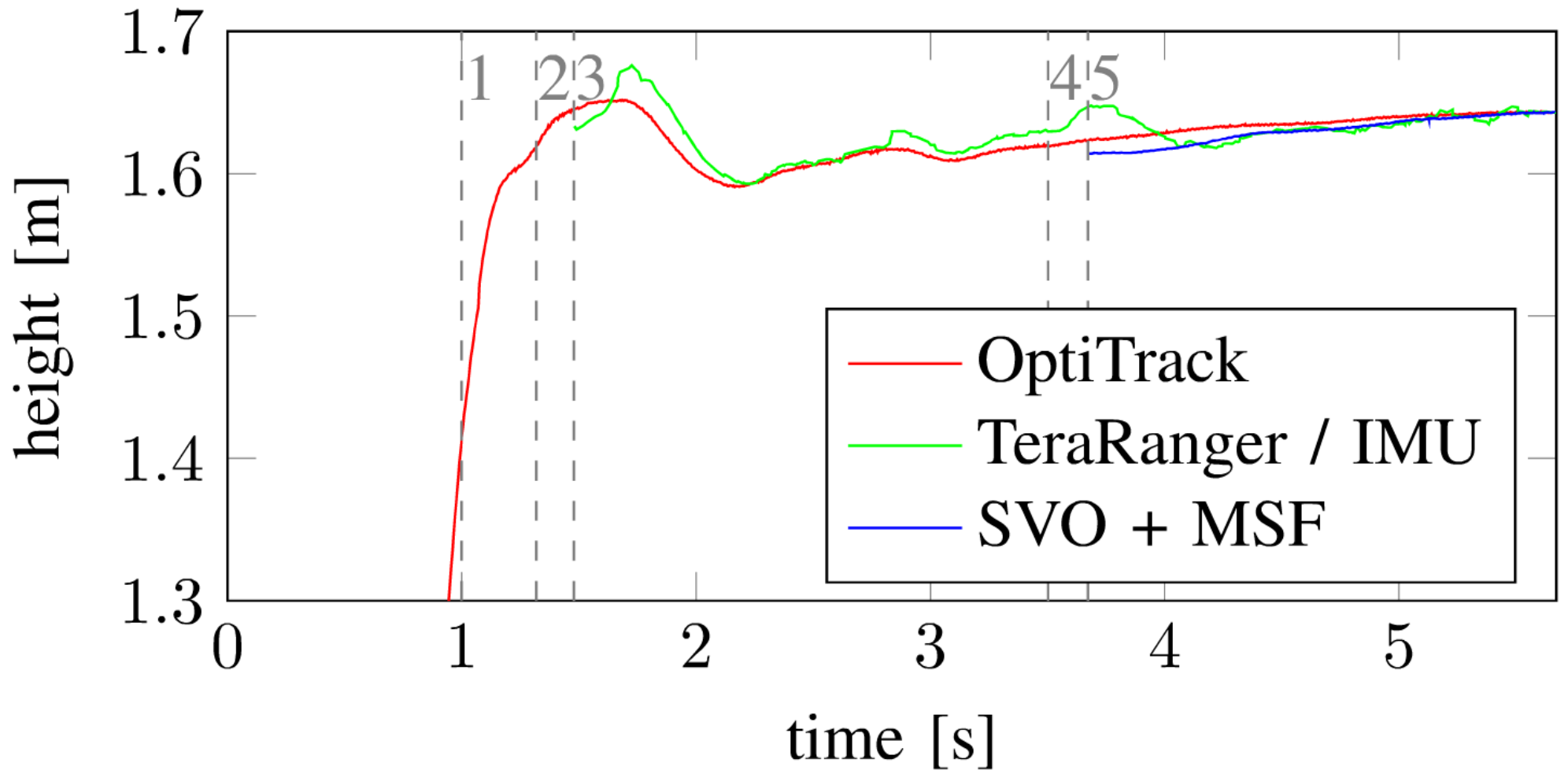
$$mc = f_1 + f_2 + f_3 + f_4$$

$$\boldsymbol{\tau} = \begin{bmatrix} \frac{\sqrt{2}}{2} l (f_1 - f_2 - f_3 + f_4) \\ \frac{\sqrt{2}}{2} l (-f_1 - f_2 + f_3 + f_4) \\ \kappa (f_1 - f_2 + f_3 - f_4) \end{bmatrix}$$

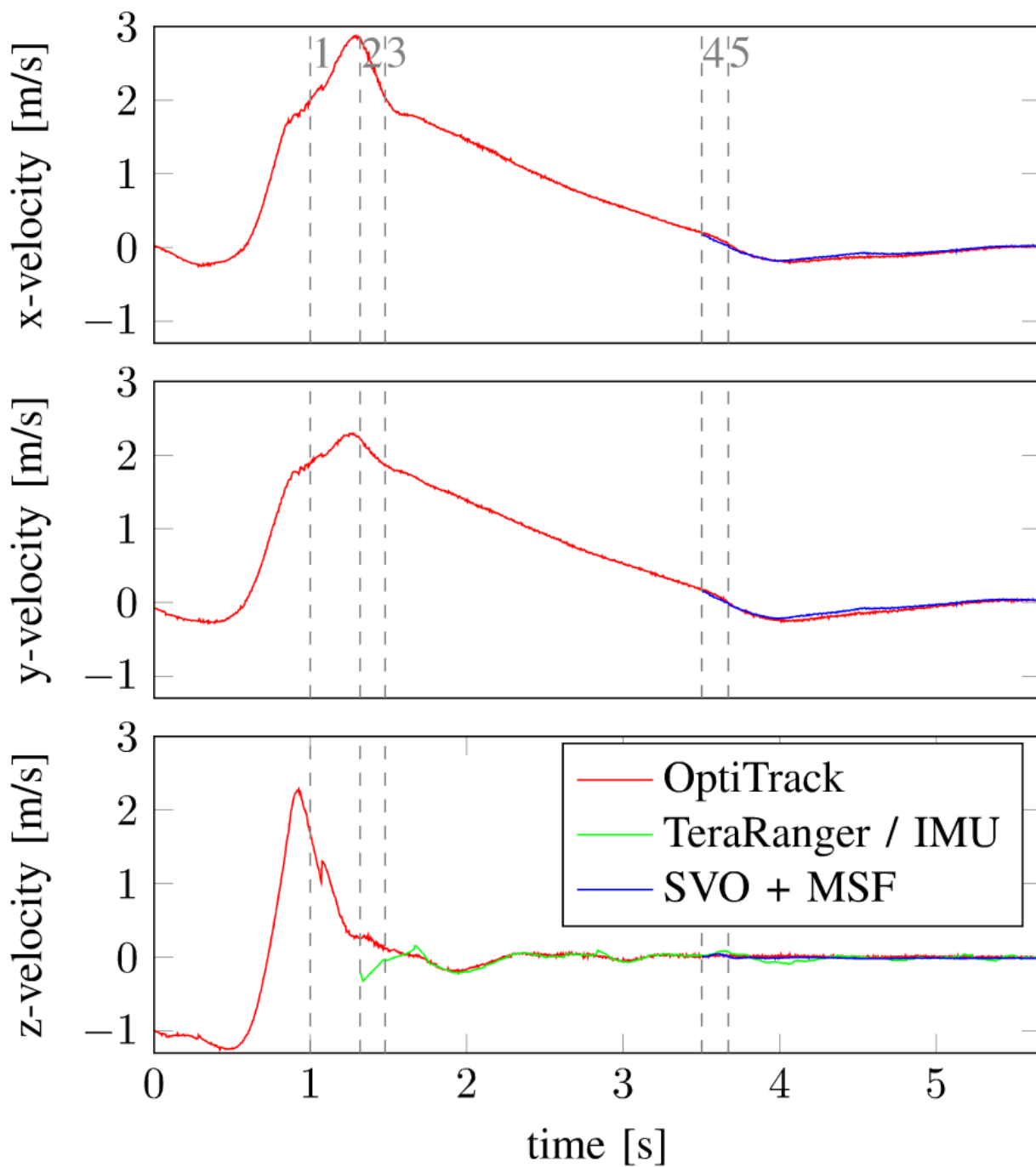
Results



Height

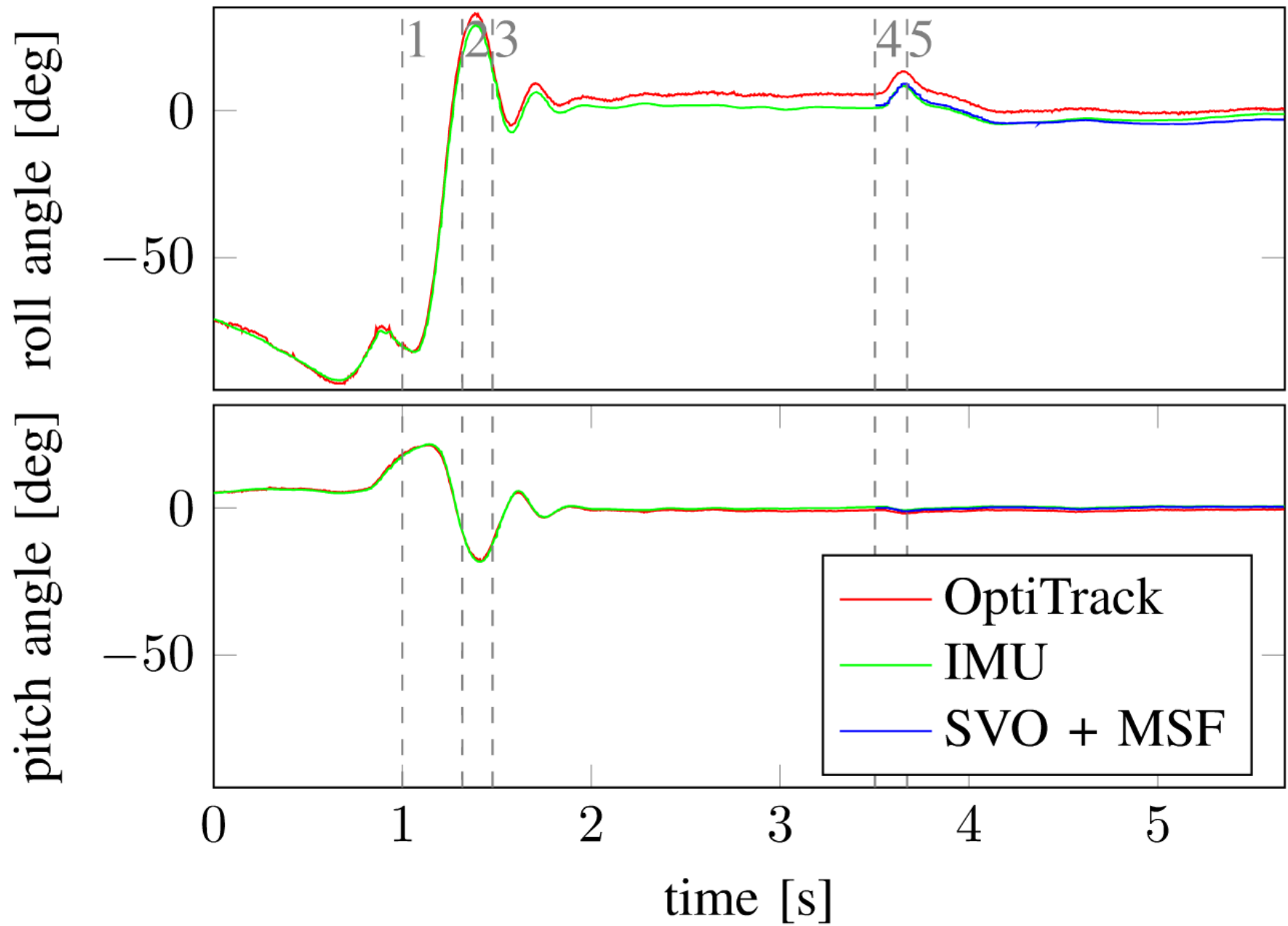


Velocity



pitch angle [deg] roll angle [deg]

Attitude



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References

- [1] C. Forster, M. Pizzoli, and D. Scaramuzza, **SVO: Fast semi-direct monocular visual odometry**, in IEEE Intl. Conf. on Robotics and Automation (ICRA), 2014.
- [2] S. Lynen, M. Achtelik, S. Weiss, M. Chli, and R. Siegwart, **A robust and modular multi-sensor fusion approach applied to MAV navigation**, in IEEE/RSJ Intl. Conf. on Intelligent Robots and Systems (IROS), 2013.
- [3] D. Brescianini, M. Hehn, and R. D'Andrea, **Nonlinear quadcopter attitude control**, Department of Mechanical and Process Engineering, ETHZ, Tech. Rep., Oct. 2013.
- [4] C. Mayhew, R. Sanfelice, and A. Teel, **Quaternion-based hybrid control for robust global attitude tracking**, Automatic Control, IEEE Transactions on, vol. 56, no. 11, pp. 2555–2566, Nov 2011.
- [5] N. Trawny and S. I. Roumeliotis, **Indirect Kalman filter for 3D attitude estimation**, University of Minnesota, Dept. of Comp. Sci. & Eng., Tech. Rep. 2005-002, Mar. 2005.
- [6] M. Faessler, F. Fontana, C. Forster, E. Mueggler, M. Pizzoli, and D. Scaramuzza, **Autonomous, vision-based flight and live dense 3D mapping with a quadrotor MAV**, J. of Field Robotics, 2015.